NOTES ON ALGEBRAIC PROPERTIES OF ROGERS SEMILATTICES

1. Preliminaries and motivation

The study of the phenomenon of computability leads to a number of very interesting directions in mathematics and applications. In recursive mathematics and computability theory, we encounter various situations which naturally lead one to the study of classes of constructive objects. An examination of the algorithmic properties of classes of constructive objects fares best with the techniques and notions of the theory of computable numberings. Arbitrary numbering of a countable class is a mapping which assigns natural indices to all elements of this class. Numbering is computable if there exists a computable function which for every object and each index of this object in numbering produces some Godel index of its constructive description. Therefore, an index of the object relative to any computable numbering can be considered as its constructive description. In the computability theory, the objects are considered modulo some computable equivalence, and the notion of equivalent numberings is the suitable notion here. Rogers semilattice of a family of sets is a quotient structure of all computable numberings of the family modulo equivalence of the numberings ordered by the relation induced by reducibility of numberings. It allows one to measure computations of a given family and are used also as a tool to classify properties of computable numberings for different families. Here, we will investigate algebraic properties of Rogers semilattices for different hierarchy.

References

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