Thermalization of Weakly Non-Integrable Many-Body Systems

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- Goals
- Proximity to integrable limits, nonintegrable networks
- Distributions and times
- Some results

Reads
PRE 95 060202(R) 2017
PRL 120 184101 2018
PRL 122 054102 2019
PRA 99 023603 2019
PRE 100 032217 2019

research and discussions with:
BL Altshuler, DK Campbell, A Cherny, C Danieli, M Fistul, Y Kati, M Thudiydangal

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Ergodicity

- Available phase space: phase space subject to constraints (e.g. energy)
- Set of points of measure one on ergodic trajectories
- Ergodic trajectory: visit proximity to any point
- Infinite time averages equal phase space averages
- Set of points of measure zero on nonergodic trajectories (e.g. periodic)

What are the different ways ergodicity may get lost ?

- Proximity to integrable limit for finite number of DoF : KAM regime
- Proximity to nonergodic set of measure zero (e.g. FPU problem)
- Additional constraints (condensation, e.g. DNLS selftrapping)
- ***Proximity to integrable limit for macroscopic number of DoF***

Goals of this talk

Ergodic dynamics of many body systems in proximity to integrable limits:

- **Integrable limit system with countable action set**
- **Identify different classes of nonintegrable perturbations**
- **Quantify ergodization process**
- **Identify novel dynamical regimes**
- **Dynamics beyond KAM for macroscopic systems at finite densities**
- **Check how canonical distributions emerge from microcanonical dynamics**

Nonergodic metallic phase near MBL phase predicted in classical limit of chains of Josephson junctions! Pino,Ioffe,Altshuler, PNAS 113, 536 (2016) (see also Escande,Kantz,Livi,Ruffo JSP 1994)

→ Dynamics of a nonintegrable system in proximity to an integrable one

Main questions

Is the system ergodic?

Do infinite time averages equal ensemble averages?

We don’t have infinite time at our disposal!

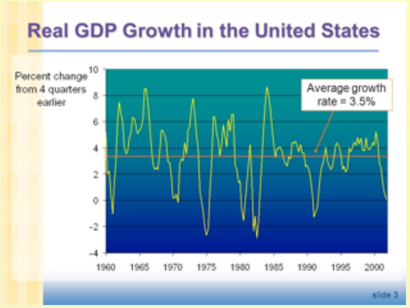
What do we know about finite time averages (FTA)?

FTA distributions?

Convergence for large averaging times?

How large is large?

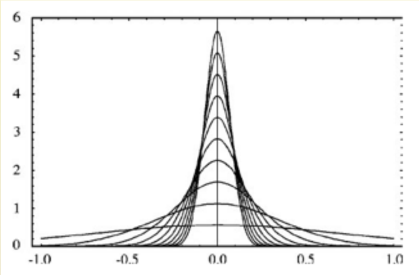
Observables f : choice and measurement



Observables f:

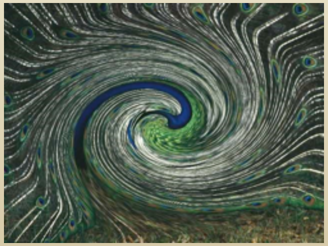
- the actions J of the corresponding integrable limit
- or some simple monotonous functions of them

Choose the right observables!



Measuring:

- f(t) = J(t)
- finite time averages (FTA) for finite averaging time T
- distributions of finite time averages
- width of distribution of FTA as function of T
- Lyapunov exponent and Lyapunov time $T_\Lambda = 1/\Lambda_{max}$



Finite time average distributions

$\frac{1}{T}\int_{\vec{R}(t=0)}^{\vec{R}(T)} f(\vec{R}(t))dt \equiv \bar{f}(T;\vec{R})$

$\rho(\bar{f},T)$: phase space distribution of $\bar{f}(T;\vec{R})$

μ_1: 1st moment of ρ (T-independent)

$\mu_2(T)$: 2nd moment of ρ (T-dependent)

$\sigma(T)=\sqrt{\mu_2(T)-\mu_1^2}$: standard deviation of ρ

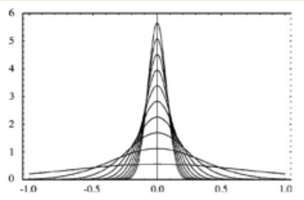
$$q(T)=\frac{\sigma^2(T)}{\mu_1^2}$$

phase space average

$\langle f\rangle \equiv \mu_1$

ergodicity:

$$\rho(\bar{f},T\to\infty)\longrightarrow\delta(\bar{f}-\langle f\rangle)$$



correlation function:

$\sigma^2(T)=\frac{1}{T}\int_0^T R_{ff}(t)dt$

$R_{ff}(t)\equiv\langle f(\tau)f(t+\tau)\rangle_\tau$

$\sigma^2(T\to\infty)\sim 1/T$?

Action spaces, network ranges

J, θ : countable action-angle set

$H = H_0(\vec{J}) + \epsilon H_1(\vec{J}, \vec{\theta})$: proximity to integrable limit if $\epsilon \ll 1$

H_1 : nonintegrable interaction network between the actions J_k

network range controls the many-body dynamics for $\epsilon \ll 1$

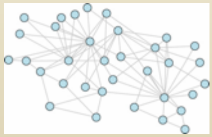
$\dot{J}_k = -\epsilon \partial H_1 / \partial \theta_k$, $\omega_k \equiv \dot{\theta}_k = \partial H / \partial J_k$

Reference action J_k coupled to R_k L_k-tuples of other actions

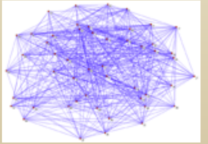
Here: $L_k = 2, 3, 4$ ... finite

Resonance: $|J_k| < \epsilon f(J_{k'} \in L_k)/\Delta\omega$

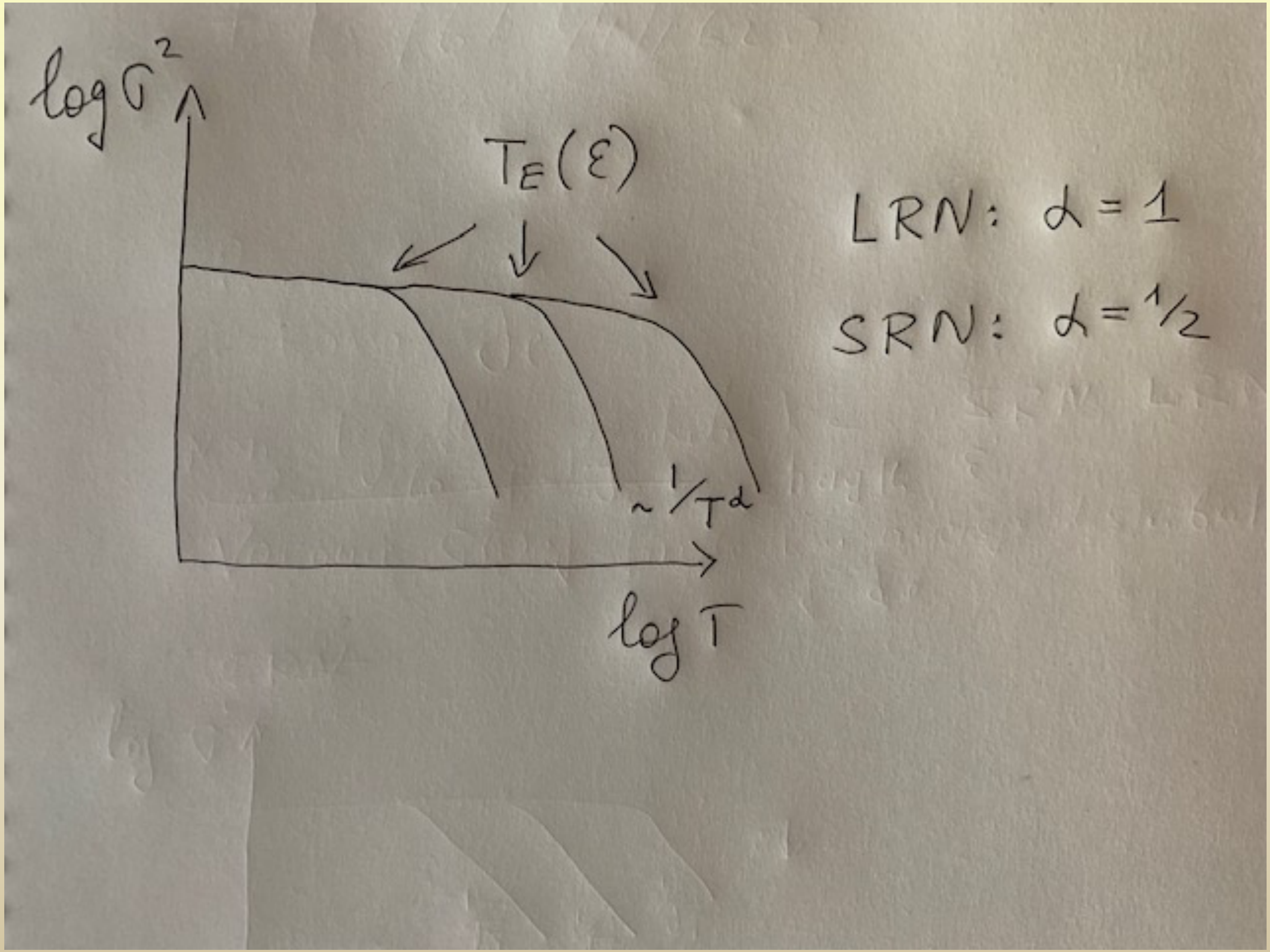
SHORT RANGE : R volume-independent
SHORT RANGE : rare chaotic resonances, slow diffusion of chaos



LONG RANGE : R monotonous increasing with volume
LONG RANGE : chaotic resonances reach everywhere: fast diffusion of chaos



A quick and incomplete summary



Josephson junction networks

$$H = \sum \frac{p_n^2}{2} + E_J[1 - \cos(q_n - q_{n-1})]$$

Number of sites (volume) : N

Energy density: h = H/N

Long Range Network

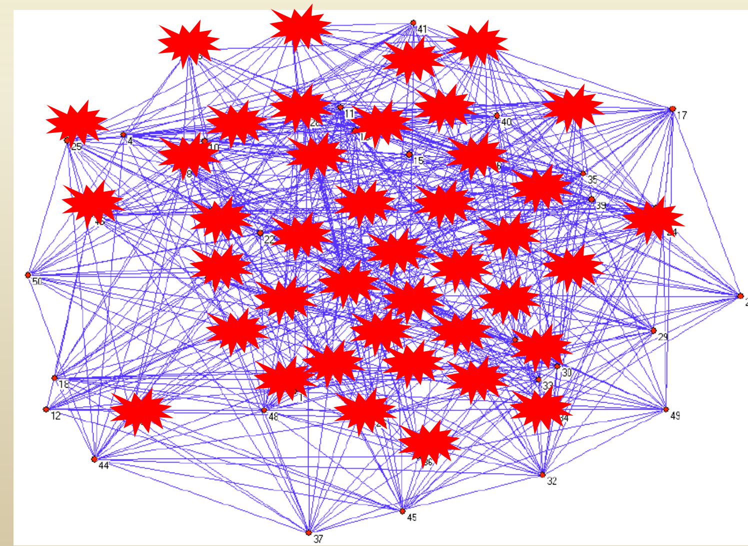
Josephson junction network, energy density h: $h/E_J \ll 1$

$$H = \sum \frac{p_n^2}{2} + E_J[1 - \cos(q_n - q_{n-1})]$$

$$H_0 = \sum \frac{p_n^2}{2} + \frac{E_J}{2}\left(q_n - q_{n-1}\right)^2 \; : \; \text{harmonic chain}$$

$$H_1 = -\frac{E_J}{4}\sum \left(q_n - q_{n-1}\right)^4 \; : \; \text{quartic anharmonicity}$$

$$L = 3, \; R \sim N^2$$



$\longrightarrow$ Long Range Network

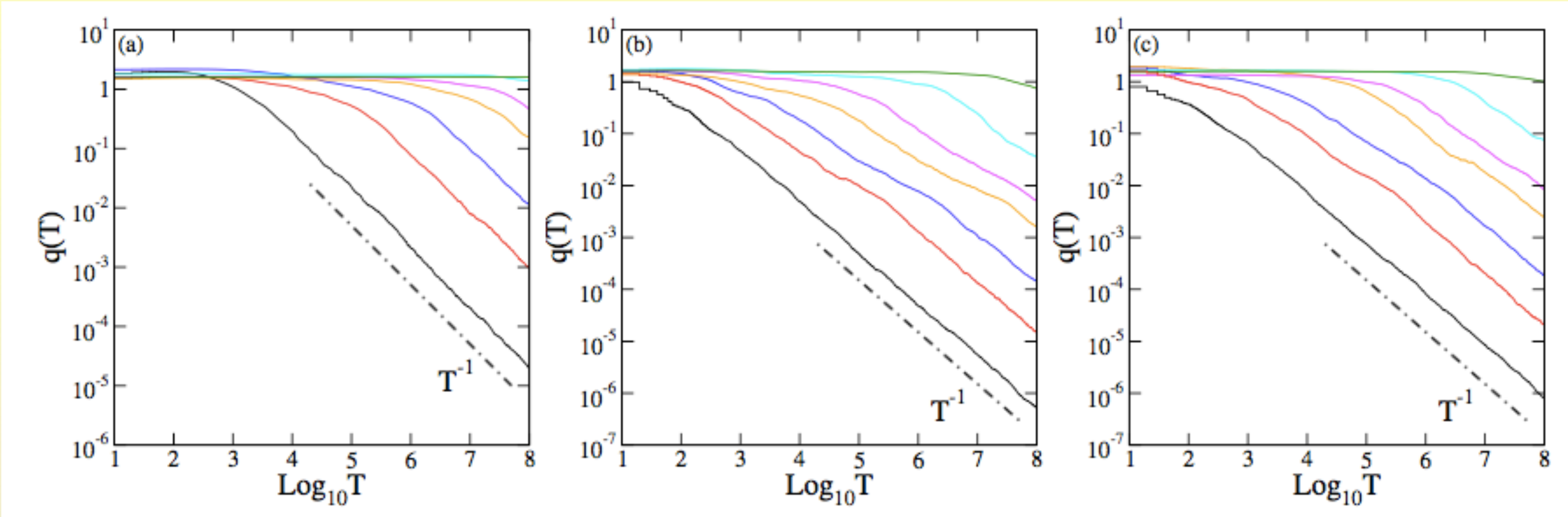
long range network:

$Q_q = \sqrt{2J_q}\sin\Theta_q \; , \; P_q = \omega_q\sqrt{2J_q}\cos\Theta_q$

$\dot{J}_q = -E_J \sum_{q_1,q_2,q_3} \omega_{q_1}\omega_{q_2}\omega_{q_3} A_{q,q_1,q_2,q_3}\sqrt{J_q J_{q_1} J_{q_2} J_{q_3}}\, cos\Theta_q \sin\Theta_{q_1} \sin\Theta_{q_2} \sin\Theta_{q_3}$

Long Range Network

disclaimer: β-FPU, N=32
(data: Danieli)



correlation function:

$$\sigma^2(T) = \frac{1}{T}\int_0^T R_{ff}(t)dt$$

$$R_{ff}(t) \equiv \langle f(\tau)f(t+\tau)\rangle_\tau$$

$$\sigma^2(T\to\infty) \sim 1/T \ \ ?$$

long range network:

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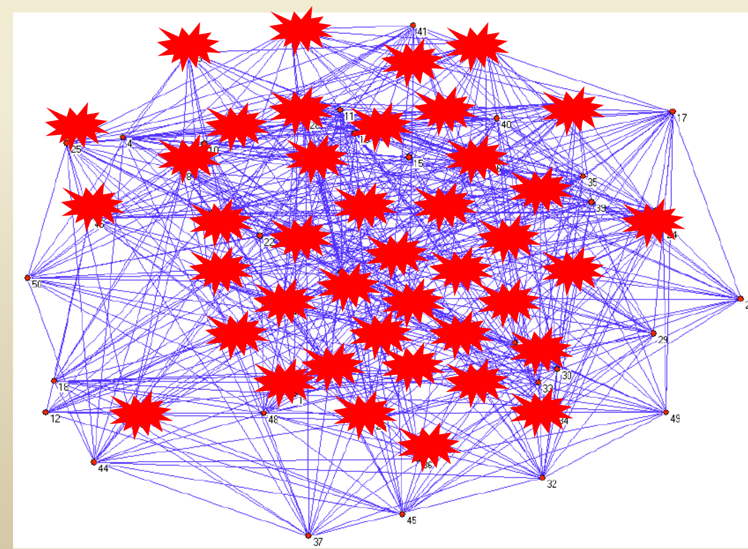


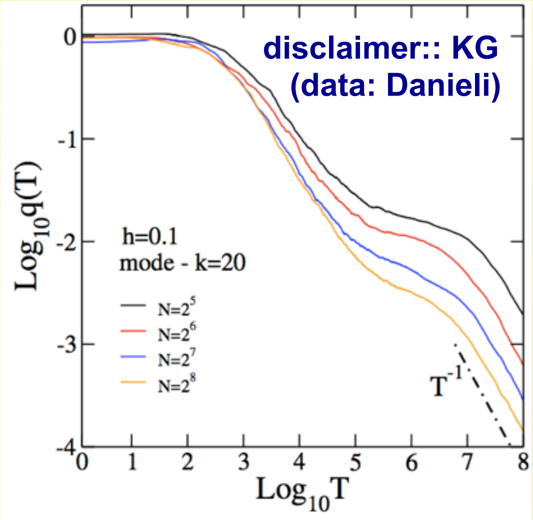
LRN: finite size corrections

long range network:

$Q_q = \sqrt{2J_q}\sin\Theta_q\ ,\ \ P_q = \omega_q\sqrt{2J_q}\cos\Theta_q$

$\dot{J}_q = -E_J\sum_{q_1,q_2,q_3}\omega_{q_1}\omega_{q_2}\omega_{q_3}A_{q,q_1,q_2,q_3}\sqrt{J_qJ_{q_1}J_{q_2}J_{q_3}}cos\Theta_q\sin\Theta_{q_1}\sin\Theta_{q_2}\sin\Theta_{q_3}$

small parameter: $\epsilon = h/E_J$
number of nonresonant actions: $N\mathrm{e}^{-N\epsilon}$
resonant action group thermalizes on time scale T_E
nonresonant action group resists thermalization up to some T_{FPUT}
nonresonant group becomes exponentially irrelevant with $N \to \infty$



$q(T) = \frac{\sigma^2(T)}{\mu_1^2}$

$q(T \ll T_E) \approx q(0) \equiv \frac{\langle J^2\rangle}{\langle J\rangle^2}$

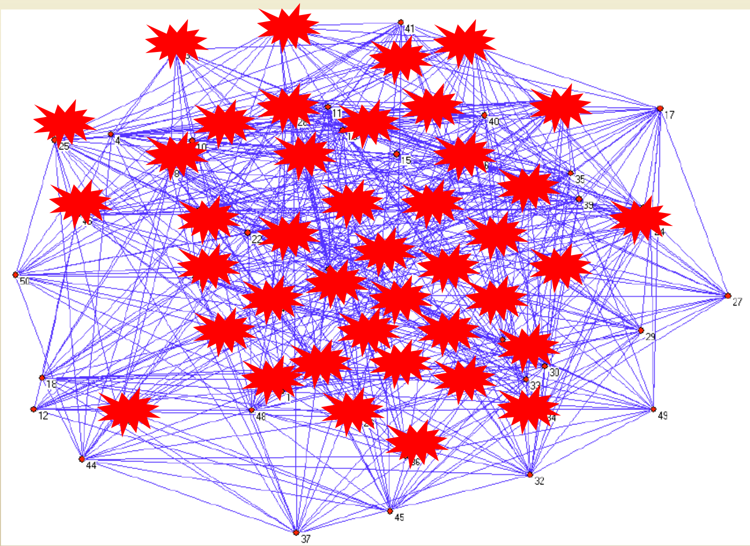
$q(T_E \ll T \ll T_{FPUT}) \approx q(0)\frac{T_E}{T}$

$q(T \sim T_{FPUT})$: shoulder !

$q(T_{FPUT} \ll T) \approx q(0)\frac{T_E}{T}$

integrable limit:

$T_E \sim T_\Lambda^2 \to \infty$



Short Range Network

Josephson junction network, $h/E_J \gg 1$

$$H = \sum \frac{p_n^2}{2} + E_J[1 - \cos(q_n - q_{n-1})]$$

$$H_0 = \sum \frac{p_n^2}{2} \ : \ \text{free rotors}$$

$$H_1 = E_J \sum [1 - \cos(q_n - q_{n-1})] \ : \ \text{nearest neighbour coupling}$$

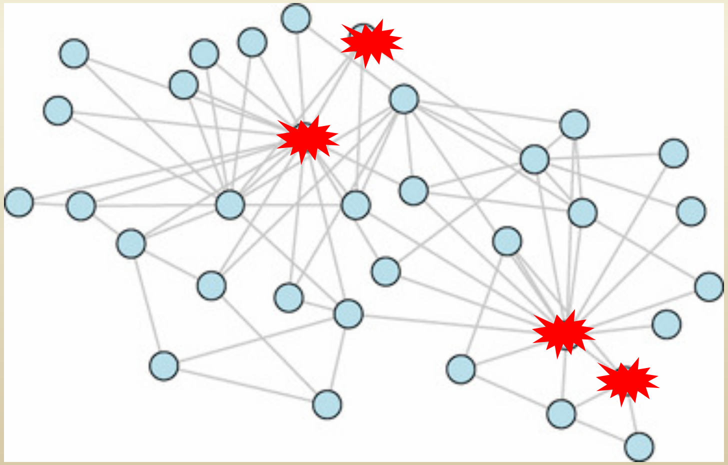
$$L = 1, \ R = 2$$

$\longrightarrow$ Short Range Network

short range network:

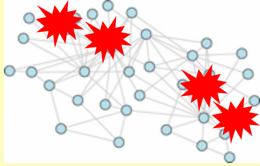
$q_n = \Theta_n \ , \ p_n = J_n$

$\dot{J}_n = -E_J \left(\sin(\Theta_n - \Theta_{n-1}) + \sin(\Theta_n - \Theta_{n+1})\right)$



SRN: resonance diffusion

short range network:



$$q_n = \Theta_n \; , \; p_n = J_n$$

$$\dot{J}_n = -E_J \left(\sin(\Theta_n - \Theta_{n-1}) + \sin(\Theta_n - \Theta_{n+1}) \right)$$

small parameter: $\epsilon = E_J/h$
resonance: $|p_{n+1} + p_{n-1} - 2p_n| < 2E_J$
distance between resonances $l_R \sim \epsilon^2$
diffusion time between resonances: $T_E \sim \epsilon^4/D$
diffusion constant: $D(\epsilon) \sim \epsilon^4/T_E$
diffusion time through system: $T_D \sim N^2/D$

$$q(T) = \frac{\sigma^2(T)}{\mu_1^2}$$

$$q(T \ll T_E) \approx q(0) \equiv \frac{\langle f^2 \rangle}{\langle f \rangle^2}$$

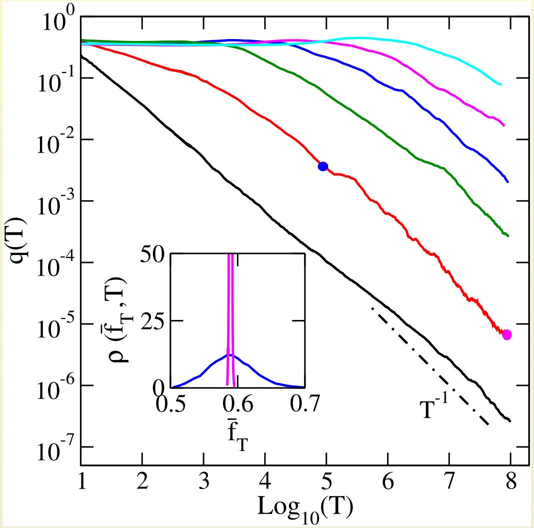
$$q(T_E \ll T \ll T_D) \approx q(0) \left(\frac{T_E}{T} \right)^{1/2}$$

$$q(T_D \ll T) \approx q(0) \frac{N}{l_R} \frac{T_E}{T}$$

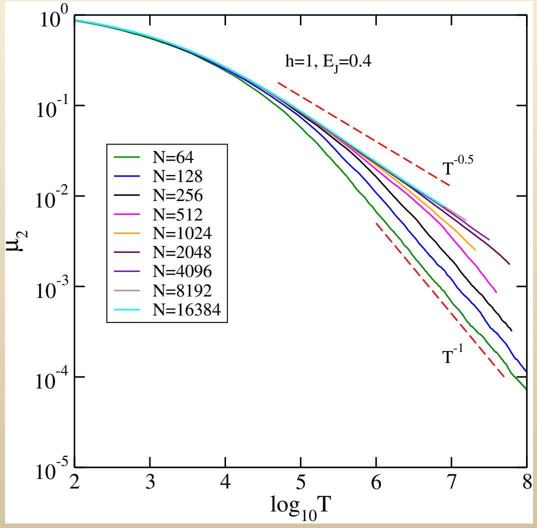
integrable limit:

$$D \to 0 \; , \; T_E \to \infty$$

f=p²/2

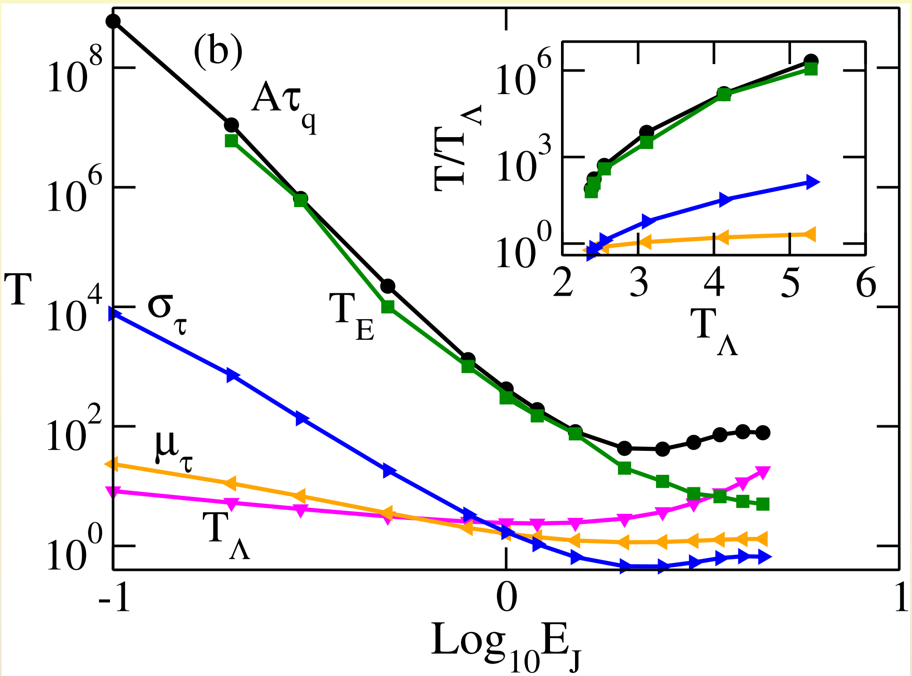


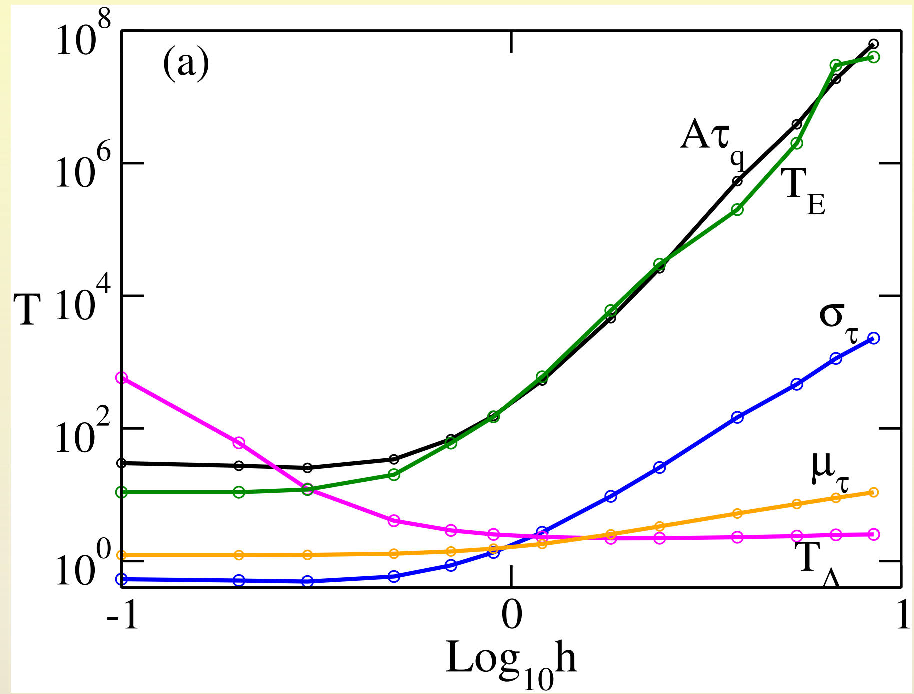
f=p



Josephson junction networks – time scales

PRL 122 054102 (2019)





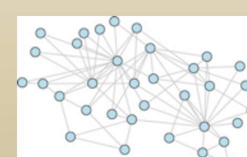
h=1
N=1024

E_J=1
N=1024

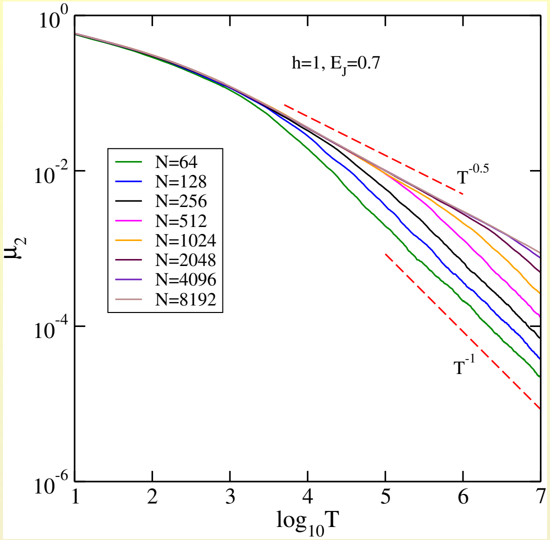
short range network:

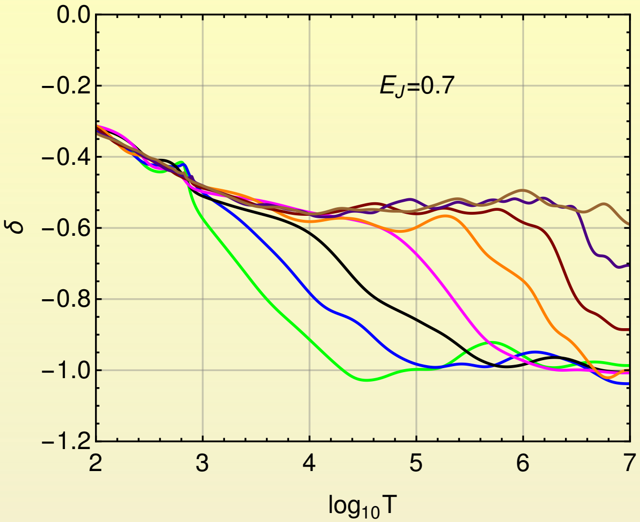
$q_n = \Theta_n \ , \ p_n = J_n$

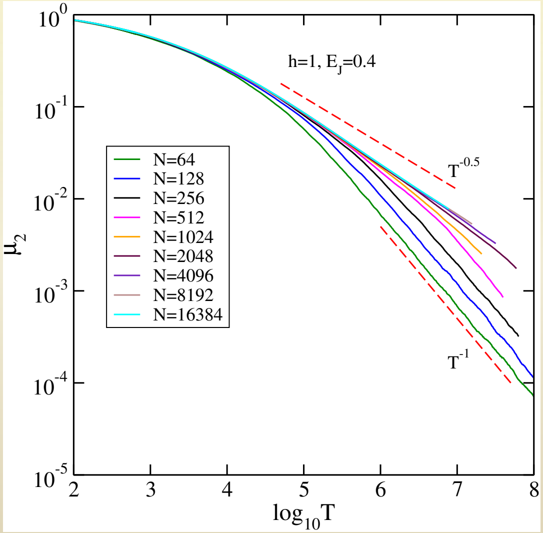
$\dot{J}_n = -E_J \left(\sin(\Theta_n - \Theta_{n-1}) + \sin(\Theta_n - \Theta_{n+1}) \right)$

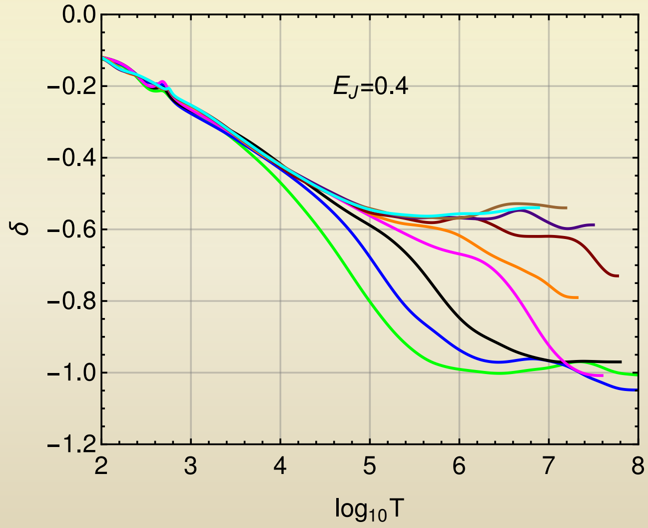


Josephson junction networks – detecting diffusion



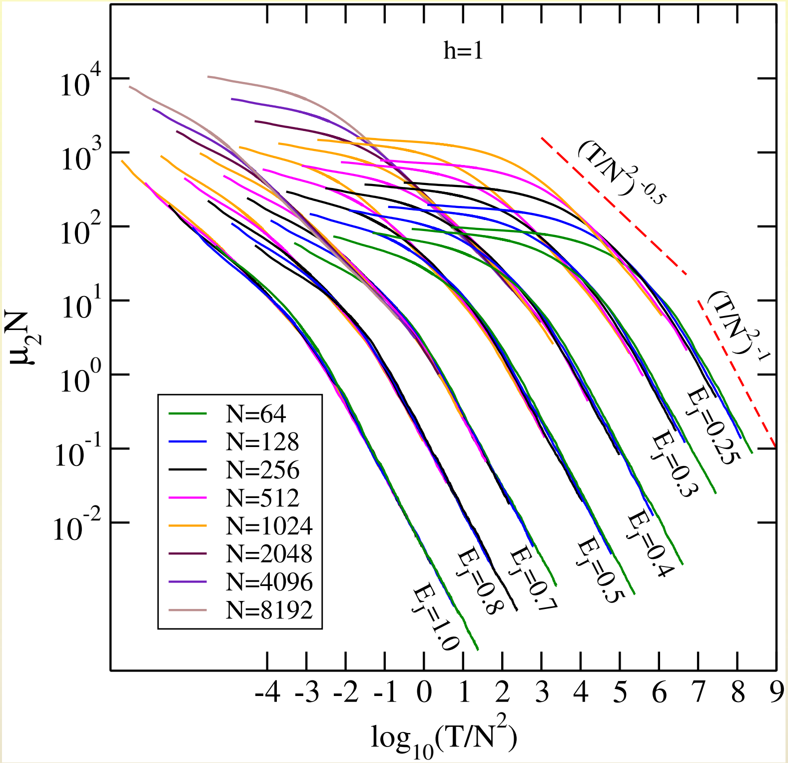


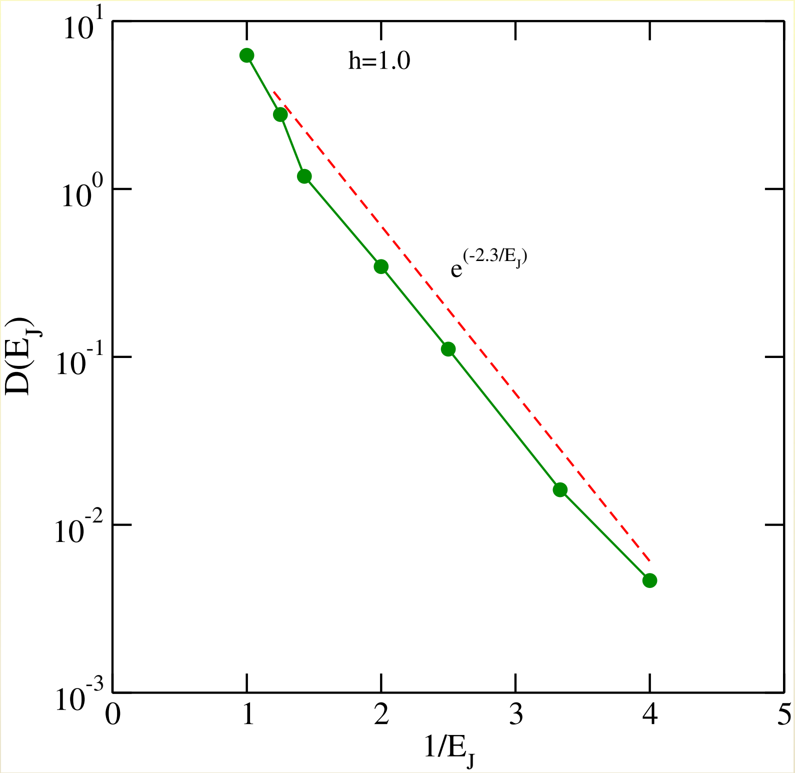




Large energy densities or small Josephson energy:
Use actions (momenta) as N ‘indistinguishable’ observables,
compute statistics of fluctuations for one observable using data from all
System size: 64,…,16384

Josephson junction networks – detecting diffusion





Large energy densities or small Josephson energy:
Use actions (momenta) as N ‘indistinguishable’ observables,
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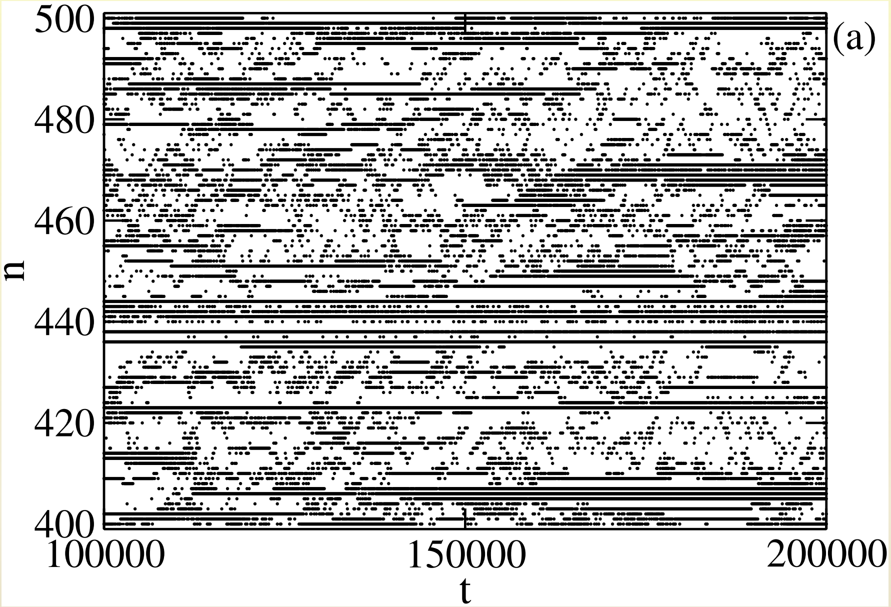
System size: 64,…,16384

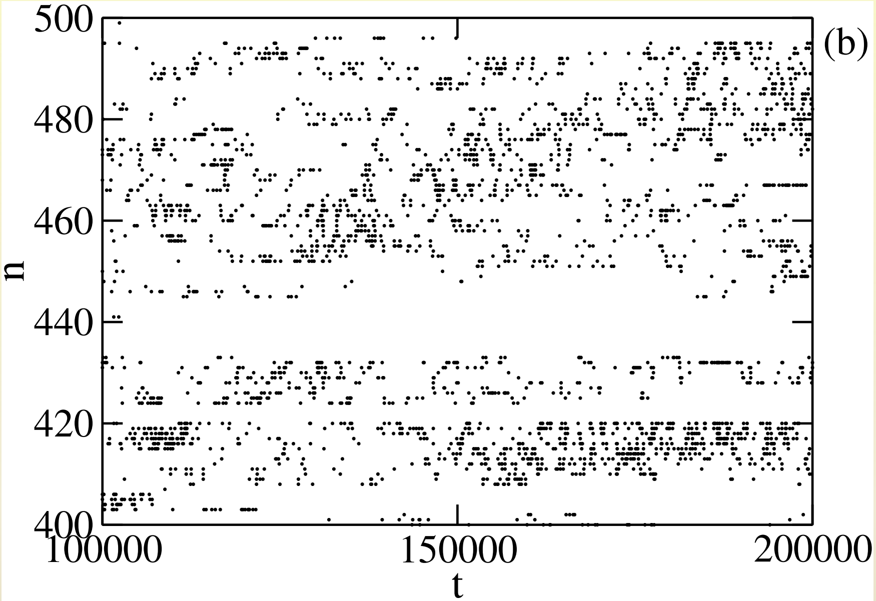
Josephson junction networks – resonance evolution

PRL 122 054102 (2019)

energy density fluctuations



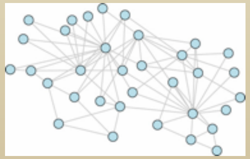
Resonance diffusion



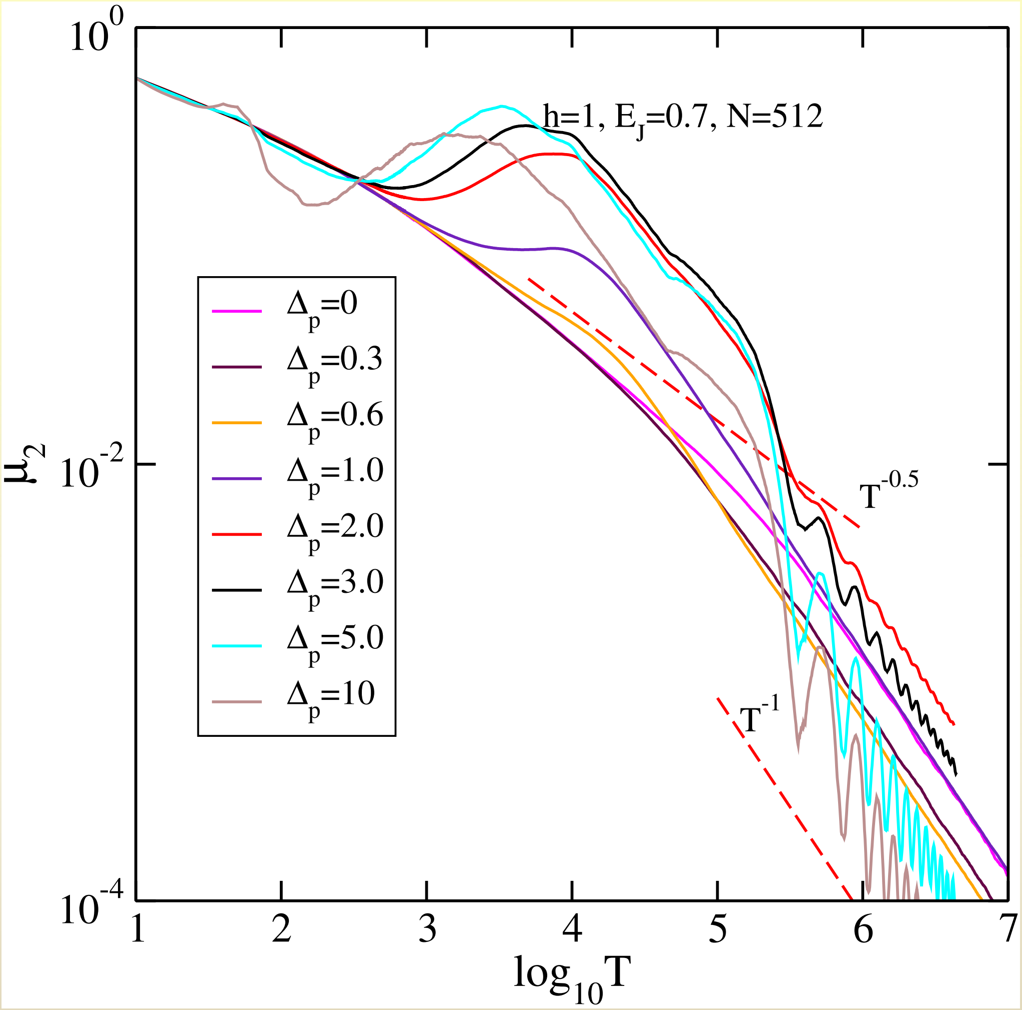
short range network:

$$q_n = \Theta_n \ , \ p_n = J_n$$

$$\dot{J}_n = -E_J \left(\sin(\Theta_n - \Theta_{n-1}) + \sin(\Theta_n - \Theta_{n+1})\right)$$



Josephson junction networks – delay diffusion by destroying resonances



Randomly choose one site every T_λ

Change momentum by $\pm\Delta_p$

Observe delay of ergodization for $\Delta_p \approx E_J$

Take Home Messages

Nonintegrable perturbations define an interaction network between actions

The network can be long or short ranged

Long range – 'nice' ergodic dynamics for large system close to integrable limit
– Lyapunov time controls ergodization time scales

Short range – transition into 'dynamical glass'
– Lyapunov time 'irrelevant' for ergodization time scales
– emerging new time scale of slow fragile resonance diffusion
– correlation functions decay algebraically
– candidate for MBL through quantization

People

Boris Altshuler (Columbia U)
David Campbell (BU)
Alexander Cherny (JINR)
Carlo Danieli (IBSPCS → MPIPKS Dresden)
Mikhail Fistul (IBSPCS)
Yagmur Kati (IBSPCS)
Mithun Thudiydangal (IBSPCS → U Mass)

Reads

PRE 95 060202(R) 2017
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Complex Condensed Matter Theory

topology, many body interactions,disorder
thermalization, glasses, degeneracy
dissipative condensates, electronic correlations
nonHermitian photonics, ultracold gases

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