Oscillation death in coupled counter-rotating identical nonlinear oscillators

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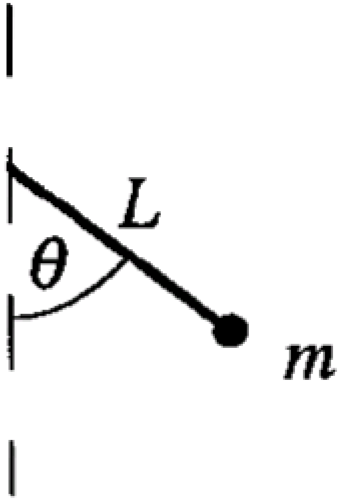
In collaboration with Woo-Sik Son (NIMS), and Dong-Uk Hwang (NIMS)

Contents

Stability of fixed point in conservative and non-conservative systems

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0$$



$$\ddot{\theta} + b\dot{\theta} + \sin\theta = 0$$

S. H. Strogatz, *Nonlinear Dynamics and Chaos* (1994)

Stability of fixed point in conservative and non-conservative systems

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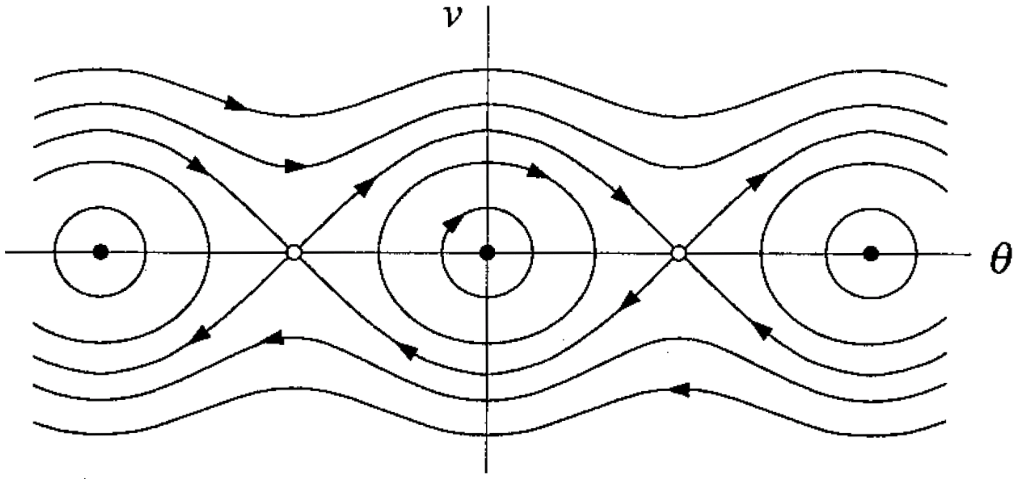

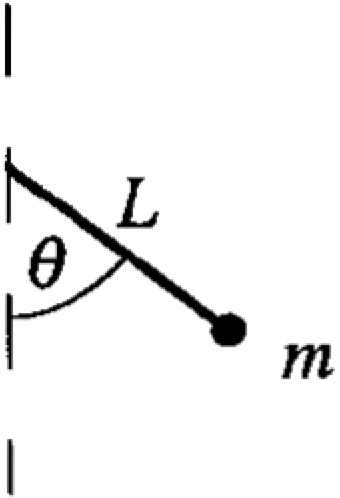

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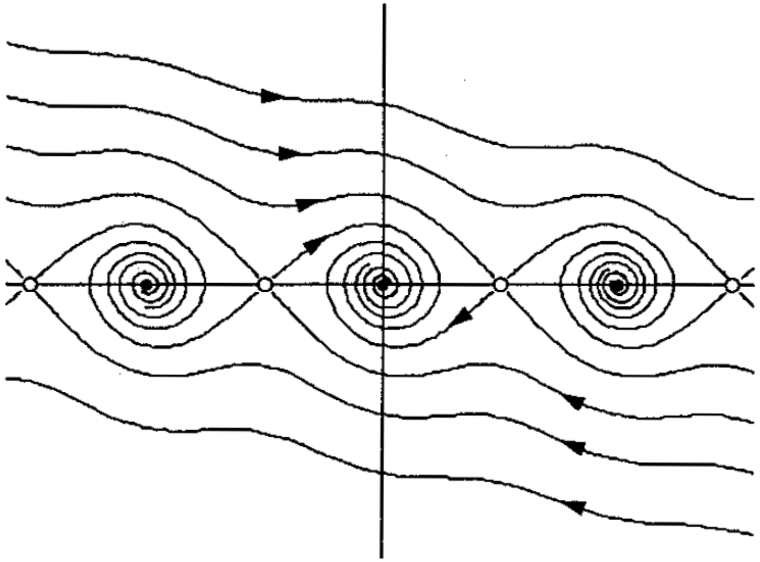

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Stability of fixed point in conservative and non-conservative systems

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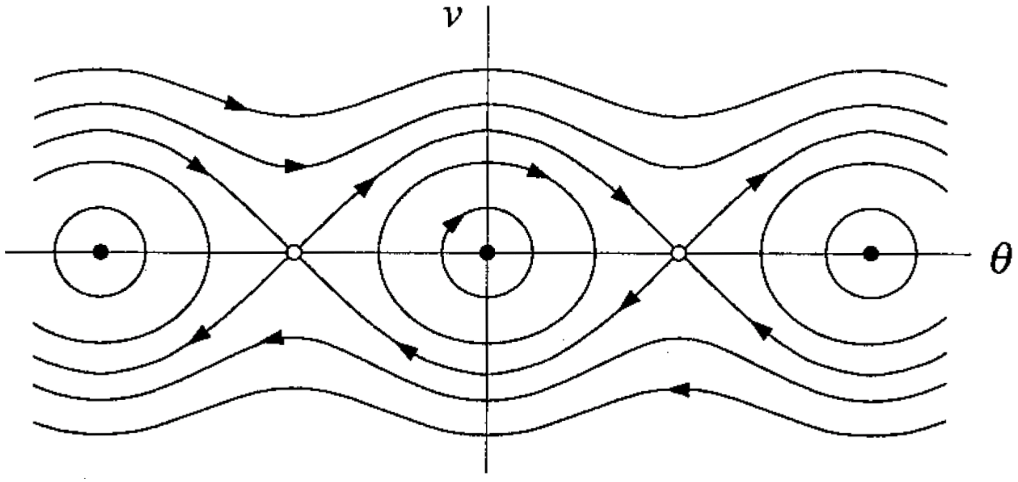

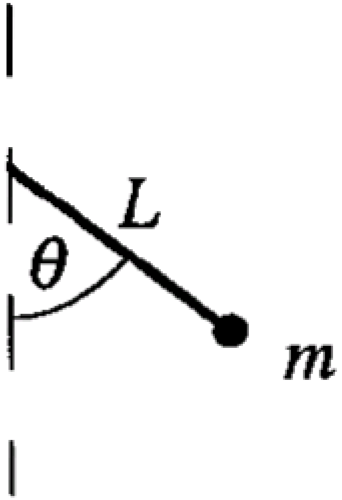

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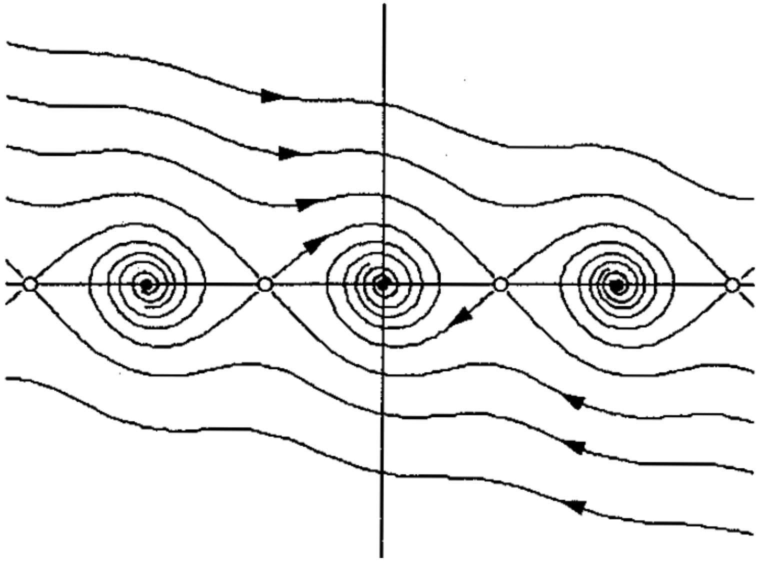
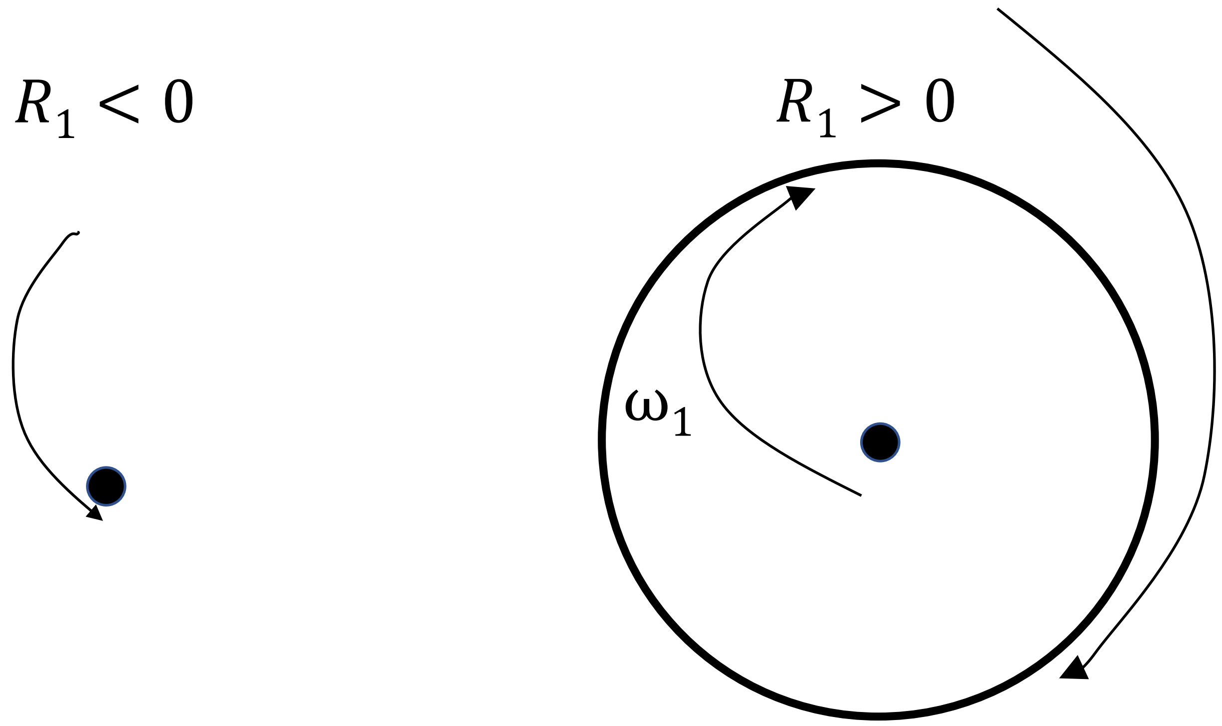


Neutrally stable or Unstable

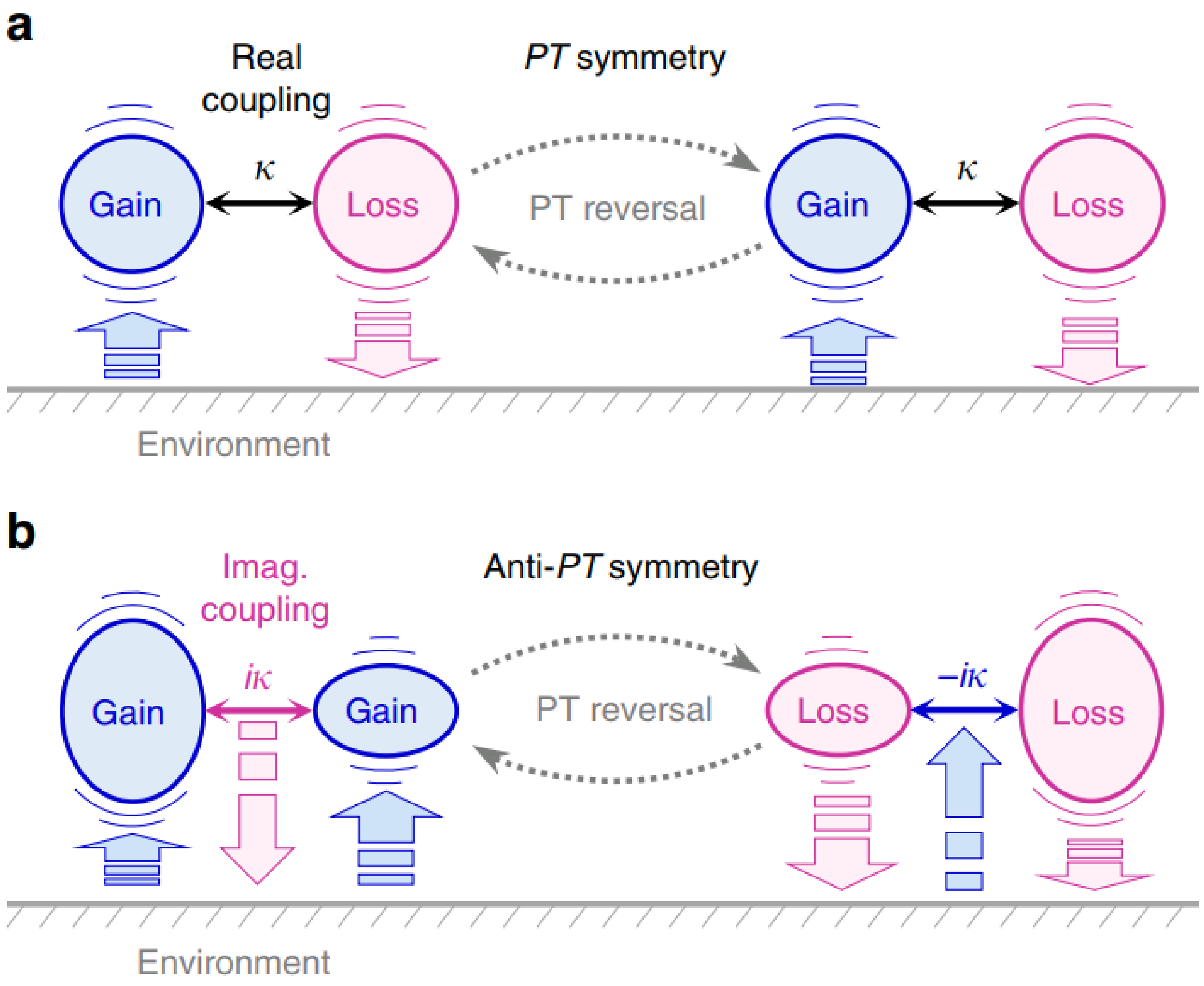
Stable or Unstable

Limit cycle oscillator: Stuart-Landau oscillator

$$\dot{z}_1 = (R_1 + i\omega_1 - |z_1|^2)z_1$$



Symmetry: Anti-PT symmetry



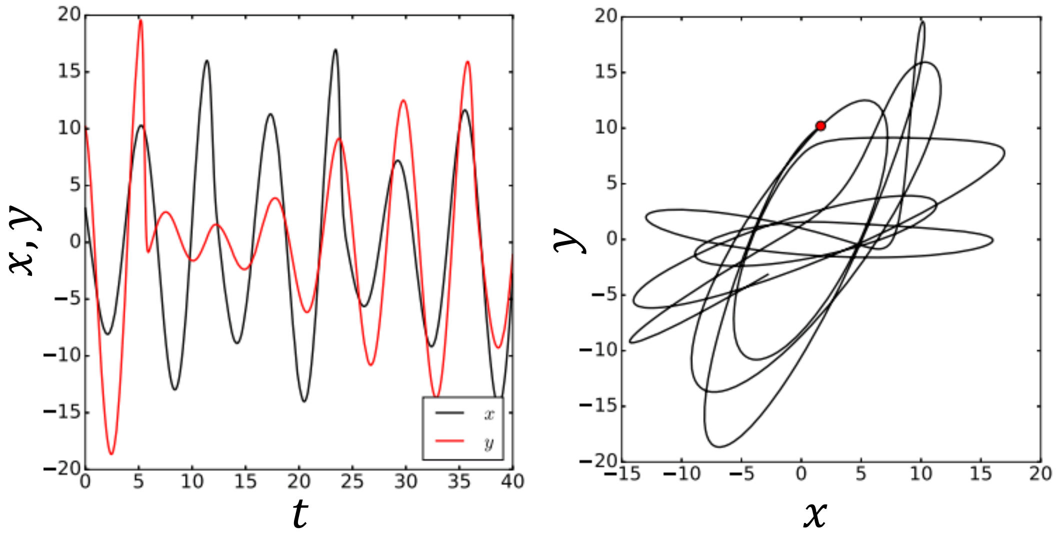
$$H^{(\mathrm{PT})} = \begin{bmatrix} \varepsilon + i\gamma & \kappa \\ \kappa & \varepsilon - i\gamma \end{bmatrix}$$

$$H^{(\mathrm{APT})} = \begin{bmatrix} -\varepsilon + i\gamma & i\kappa \\ i\kappa & \varepsilon + i\gamma \end{bmatrix}$$

Choi et al., Nat. Commun. 9, 2182 (2018)

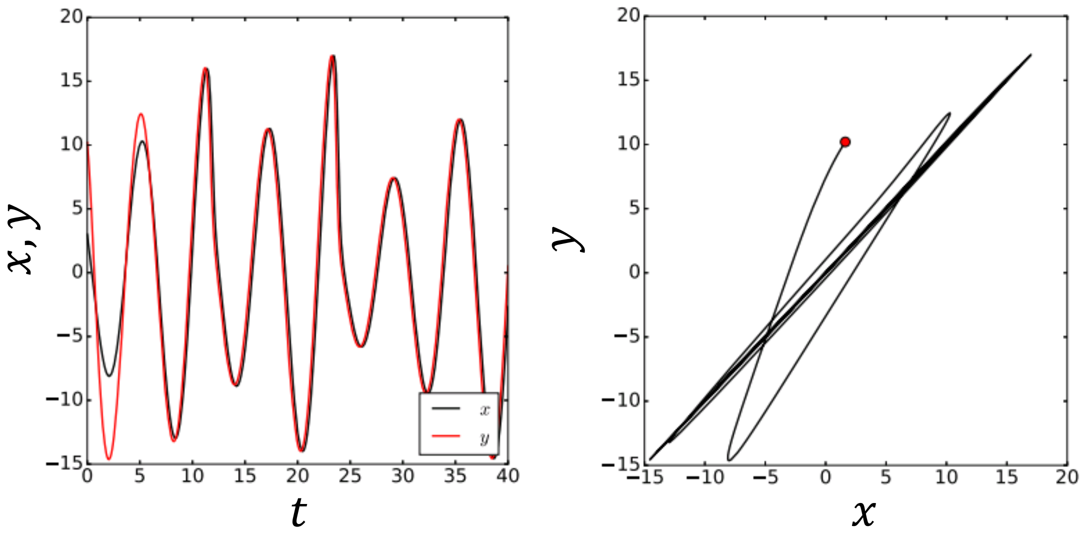
Complete synchronization of coupled identical chaotic oscillators

$$\dot{x} = F(x) + a(y - x)$$
$$\dot{y} = F(y) + a(x - y)$$



$x(t) \neq y(t)$

nonsynchronization: symmetry broken steady state

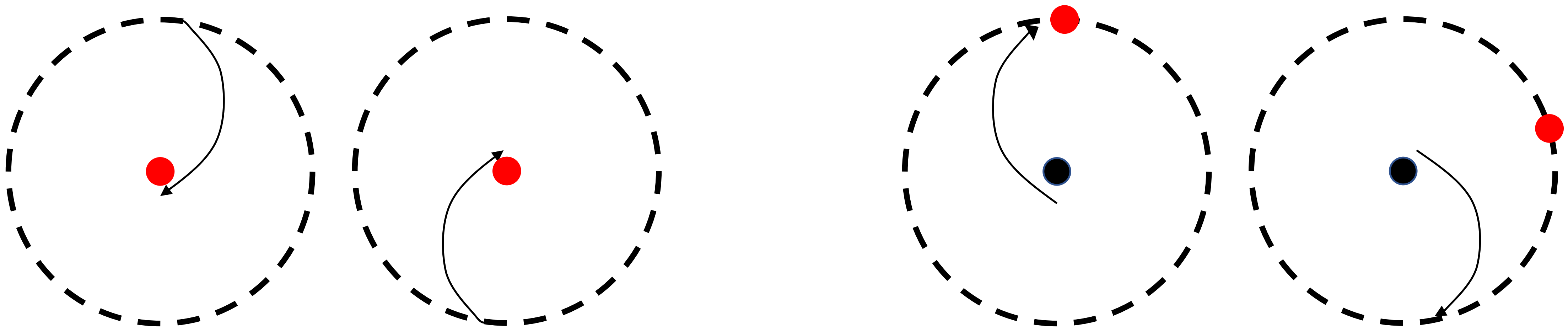


$x(t) = y(t)$

Synchronization: symmetric steady state

Pikovsky et al., Synchronization: A Universal Concept in Nonlinear Sciences (2001)

Oscillation quenching mechanisms: Amplitude vs. oscillation death



Amplitude death
– homogeneous steady states

Oscillation death
– inhomogeneous steady state

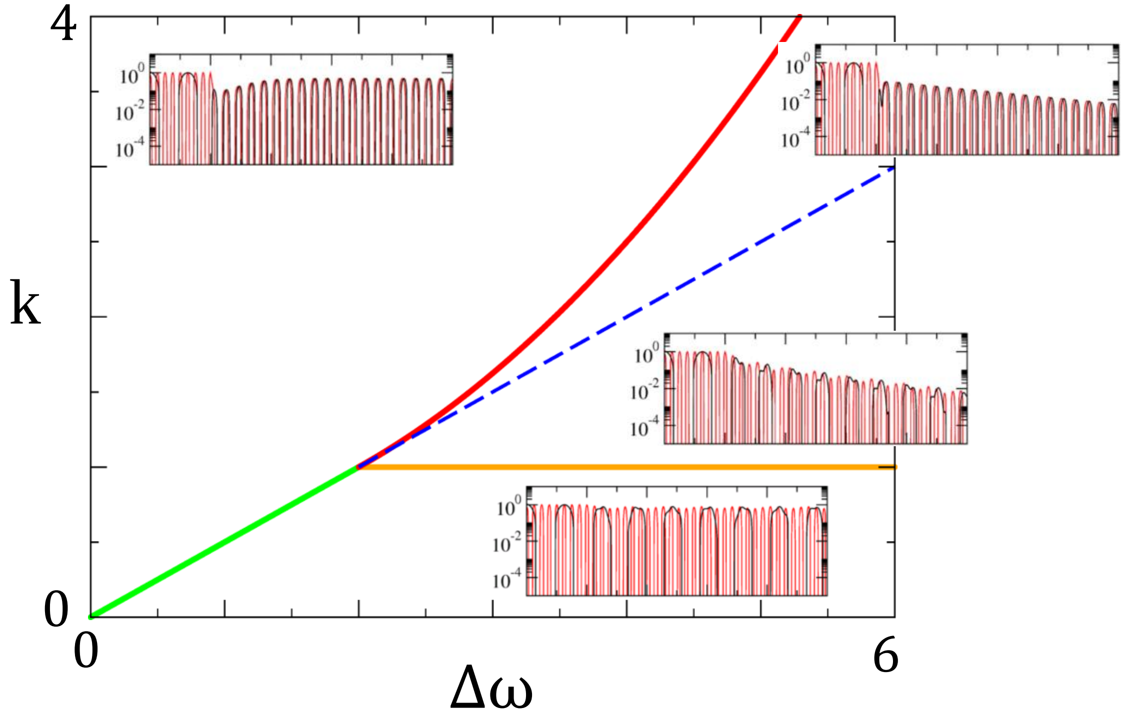
Koseska et al., Physics Reports 531 (2013)

Coupled counter-rotating identical Stuart-Landau oscillators

Coupled co-rotating non-identical Stuart-Landau oscillators

$$\dot{z}_1 = (R_1 + i\omega_1 - |z_1|^2)z_1 + k(z_2 - z_1),$$

$$\dot{z}_2 = (R_2 + i\omega_2 - |z_2|^2)z_2 + k(z_1 - z_2),$$



Ryu et al., Phys. Rev. E 91, 052910 (2015)

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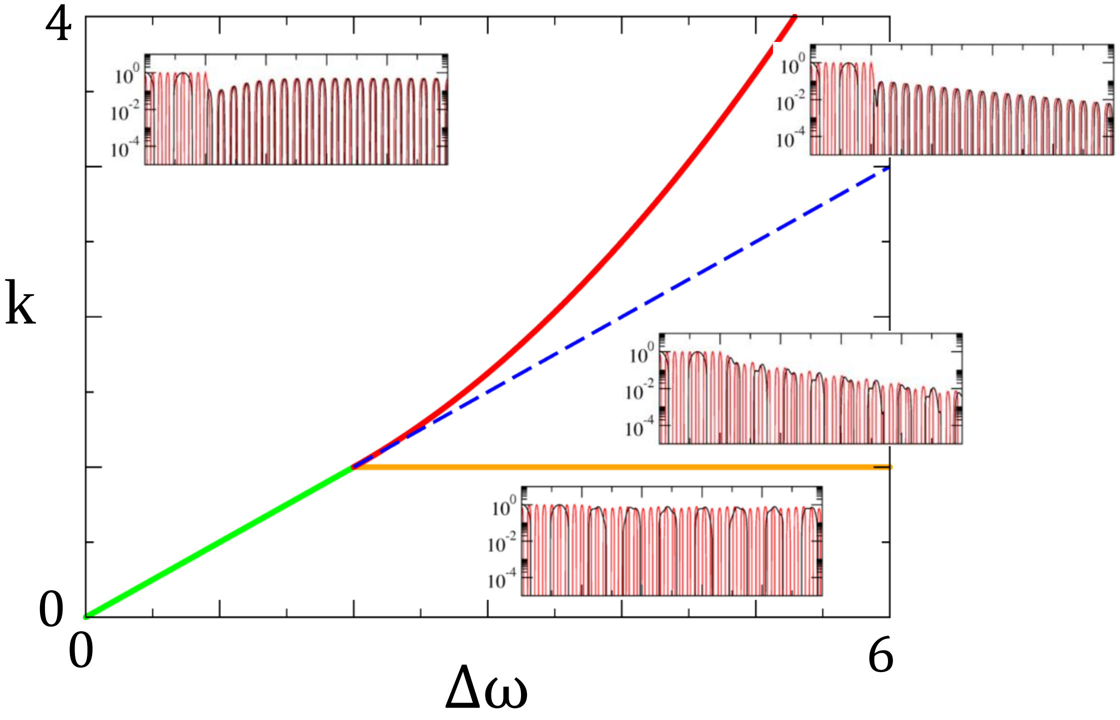

$$i\frac{d}{dt}\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \mathrm{H_{eff}}\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

$$\mathrm{H_{eff}} = \begin{pmatrix} -\omega_1 + i(1-k) & ik \\ ik & -\omega_2 + i(1-k) \end{pmatrix}$$

+ Anti-PT-symmetry

$$H^{(\mathrm{APT})} = \begin{bmatrix} -\varepsilon + i\gamma & i\kappa \\ i\kappa & \varepsilon + i\gamma \end{bmatrix}$$

$$\omega_2 = -\omega_1$$

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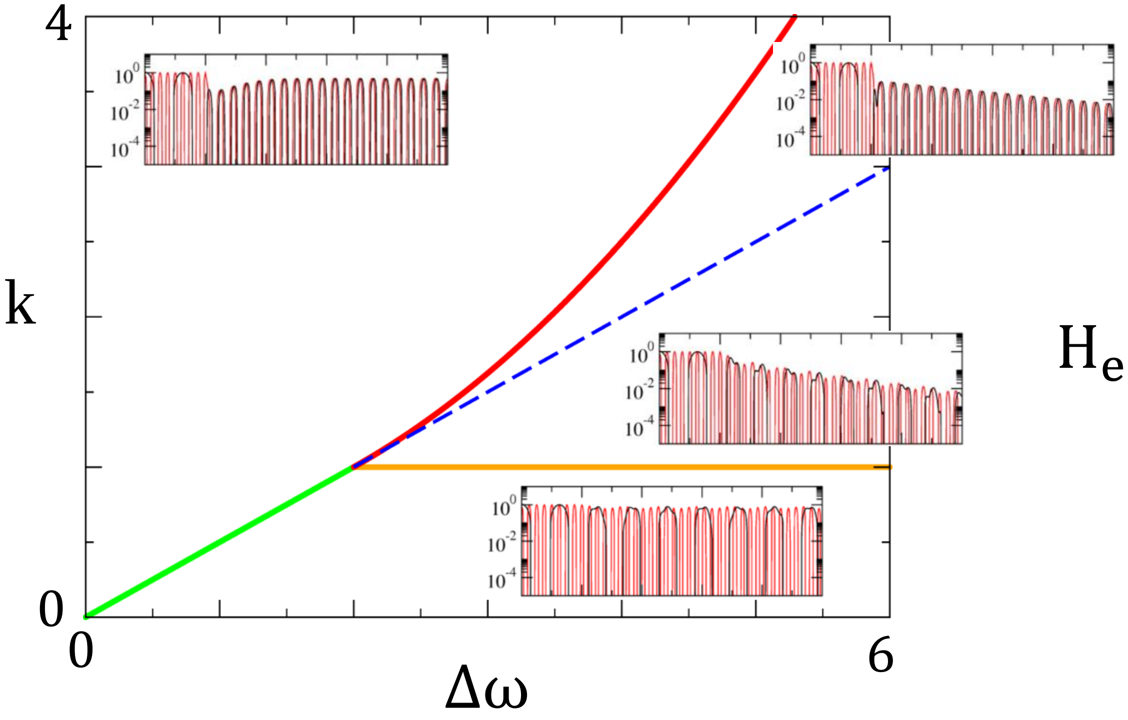

Coupled counter-rotating identical Stuart-Landau oscillators

$$\dot{z}_1 = (1 + i\omega - |z_1|^2)z_1 + k(z_2 - z_1),$$

$$\dot{z}_2 = (1 - i\omega - |z_2|^2)z_2 + k(z_1 - z_2),$$

$$H_{\text{eff}} = M = iJ = \begin{pmatrix} \omega + i(1-k) & ik \\ ik & -\omega + i(1-k) \end{pmatrix}$$

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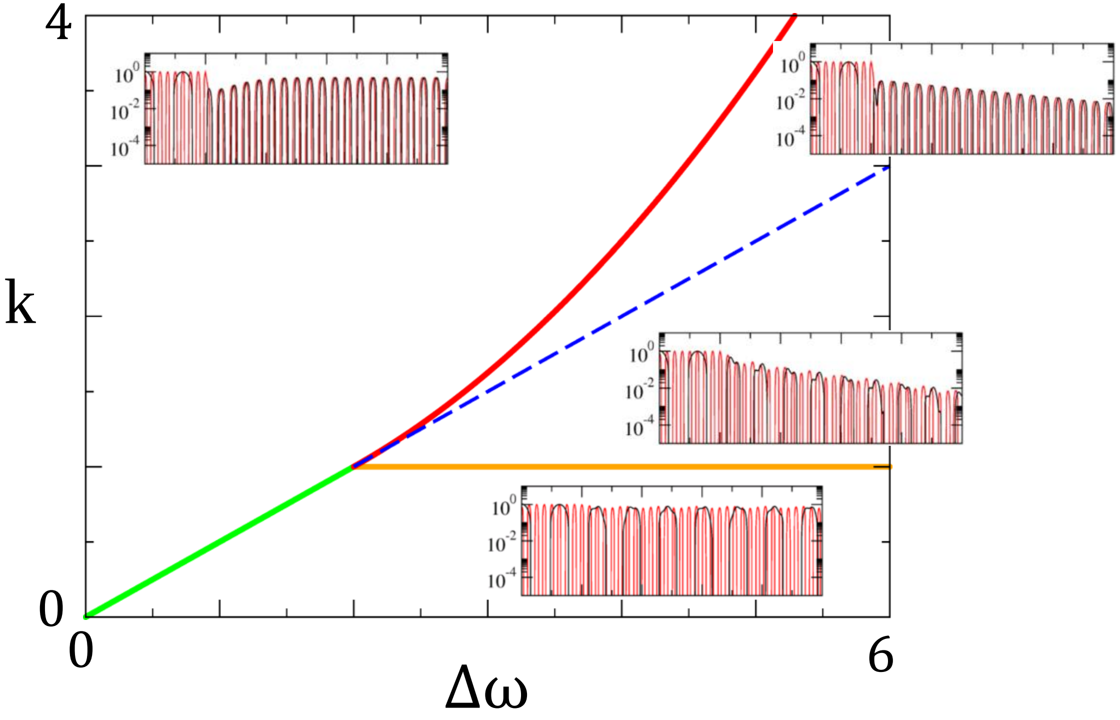

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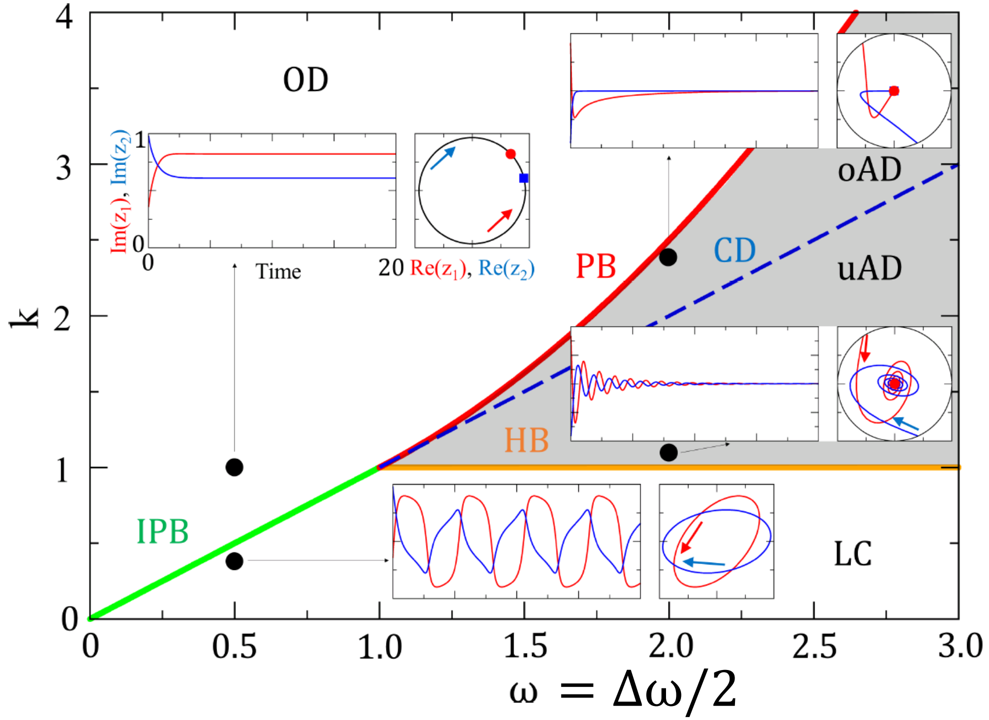

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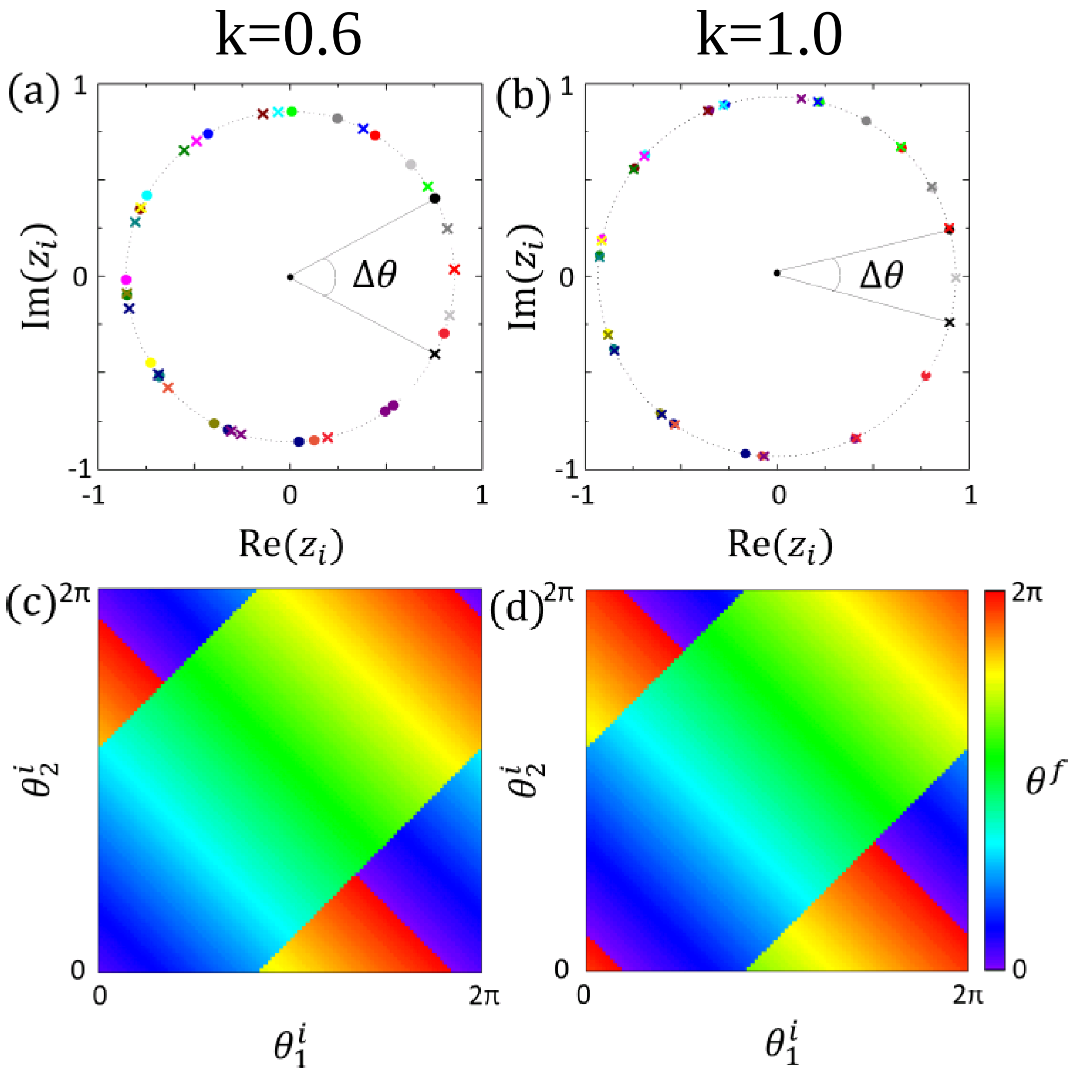
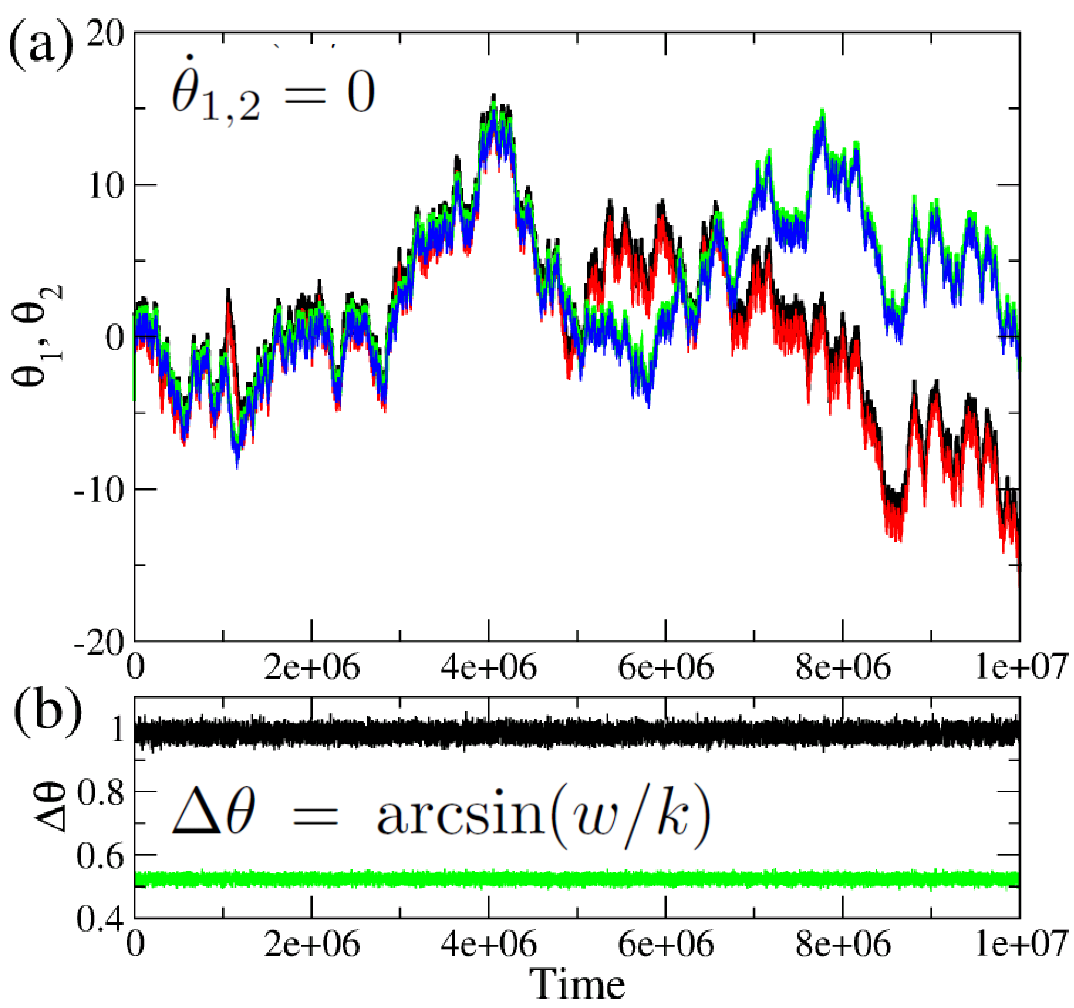


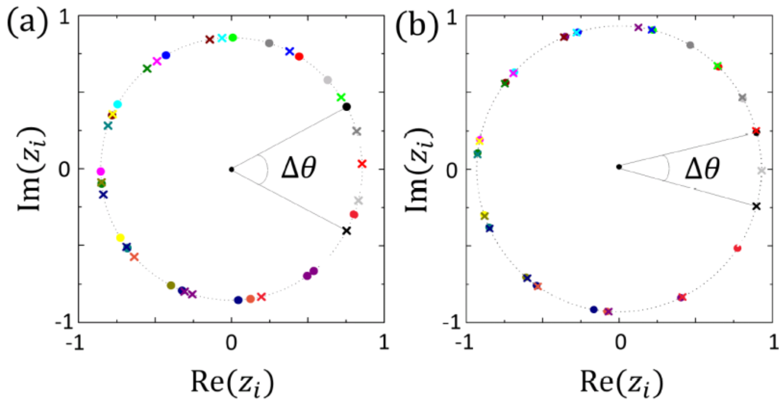
FIG. 2: (color online) The final steady states of oscillation death when (a) $k = 0.6$ and (b) $k = 1.0$ with $\omega = 0.5$. The circles and crosses denote the steady states of first and second oscillators. The different color represents the different initial points. The angular phases of final states θ^f of (c) first and (d) second oscillators as a function of the initial angular phases $\theta^i_{1,2}$.

Noise effect on neutrally stable OD states



$$\dot{z}_1 = (1 + i\omega - |z_1|^2)z_1 + k(z_2 - z_1) + h\xi_1,$$
$$\dot{z}_2 = (1 - i\omega - |z_2|^2)z_2 + k(z_1 - z_2) + h\xi_2,$$

FIG. 4: (color online) (a) Angular phases of two oscillators when $k = 0.6$ and $k = 1.0$ with $\omega = 0.5$. Black and red trajectories represent angular phases of first and second oscillators, respectively, when $k = 0.6$. Green and blue trajectories represent angular phases of first and second oscillators, respectively, when $k = 1.0$. (b) The phase difference between two oscillators when $k = 0.6$ (black) and $k = 1.0$ (green), respectively.



Experimental Implementation

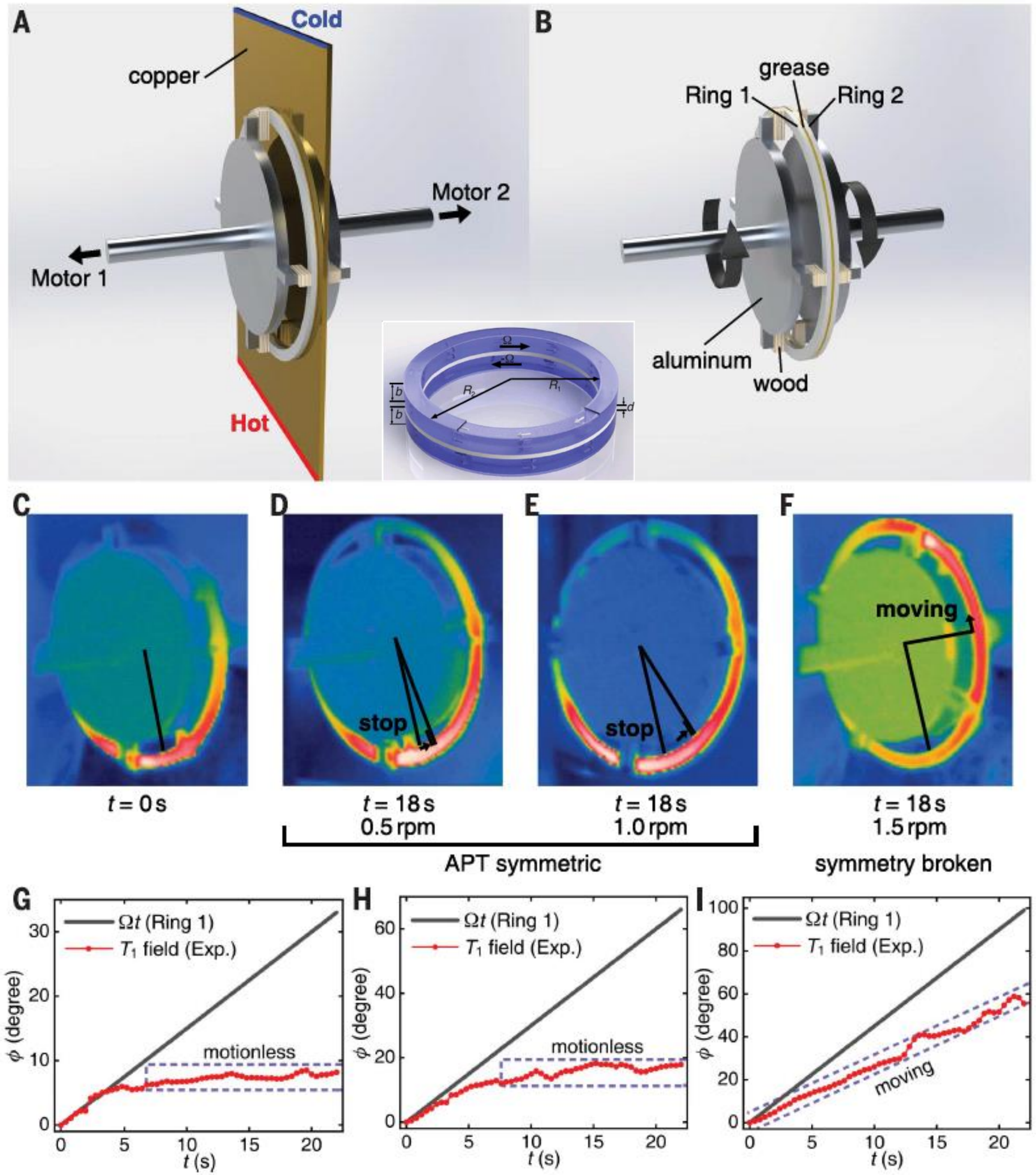
PHYSICS

Anti–parity-time symmetry in diffusive systems

Ying Li[1]*, Yu-Gui Peng[1,2]*, Lei Han[1,3]*, Mohammad-Ali Miri[4,5], Wei Li[6], Meng Xiao[6,7], Xue-Feng Zhu[2]†, Jianlin Zhao[3], Andrea Alù[4,8,9,10], Shanhui Fan[6]†, Cheng-Wei Qiu[1]†

Various concepts related to parity-time symmetry, including anti–parity-time symmetry, have found broad applications in wave physics. Wave systems are fundamentally described by Hermitian operators, whereas their unusual properties are introduced by incorporation of gain and loss. We propose that the related physics need not be restricted to wave dynamics, and we consider systems described by diffusive dynamics. We study the heat transfer in two countermoving media and show that this system exhibits anti–parity-time symmetry. The spontaneous symmetry breaking results in a phase transition from motionless temperature profiles, despite the mechanical motion of the background, to moving temperature profiles. Our results extend the concepts of parity-time symmetry beyond wave physics and may offer opportunities to manipulate heat and mass transport.

Li et al., Science 364, 170–173 (2019)



Summary and Discussions

- We have found the new type of oscillation suppression in coupled counter-rotating identical nonlinear oscillators, of which steady states are neutrally stable.

- The neutral stability of the oscillation death is originated from the anti-PT-symmetry of the systems.

- We expect that new emergent states related to the conservative properties such as neutral stability in dissipative nonlinear systems can be generated by the symmetry recovered by spontaneous symmetry breaking of PT-symmetry such as anti-PT-symmetry of this work.

- *Ryu et al., Phys. Rev. E 100, 022209 (2019).*

Coupled counter-rotating identical Stuart-Landau oscillators

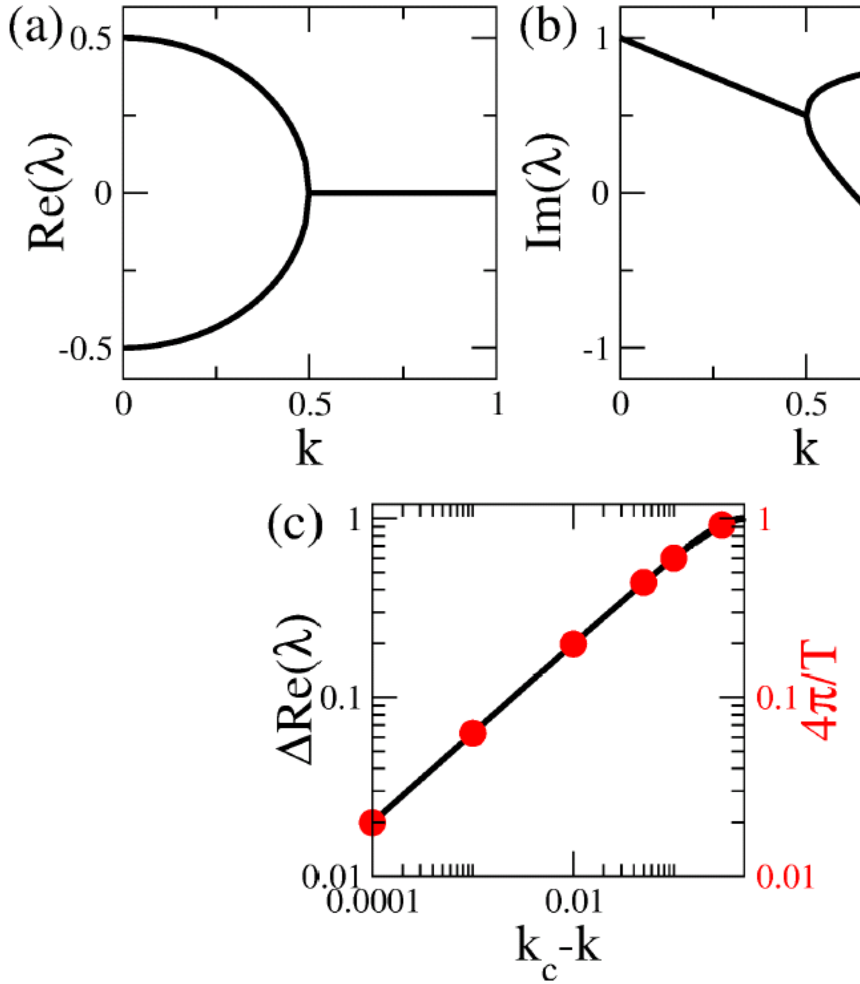


FIG. 3: (color online) (a) Real and (b) imaginary parts of two eigenvalues of M as a function of k when $\omega = 0.5$. EP occurs when $k = 0.5$. (c) Difference between two real parts (black line) and periods (red circles) of the oscillations as a function of k.