Nondegenerate and degenerate solitons in certain physically important integrable coupled Nonlinear Schrödinger Equations

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Plan of talk

1. Manakov system & other coupled NLS equations
2. Manakov system: Fundamental vector bright (degenerate) solitons
3. Vector bright solitons in mixed coupled NLS and similar systems
4. Nondegenerate fundamental solitons in Manakov system
5. Collision properties of nondegenerate solitons
6. Nondegenerate solitons in other coupled NLS equations
7. Conclusions

Manakov System

Optical pulse propagation in two mode/birefringent fiber

$$iq_{1,z} + q_{1,tt} + 2(|q_1|^2 + |q_2|^2)q_1 = 0$$
$$iq_{2,z} + q_{2,tt} + 2(|q_1|^2 + |q_2|^2)q_2 = 0 \quad (1)$$

- Manakov introduced the above coupled equation for orthogonally polarized optical waves and obtained two-soliton solution using Inverse Scattering Transform method.
- Completely integrable Hamiltonian system: Lax Pair, N-bright soliton solution, Infinite number of conserved quantities.

S. V. Manakov, Sov. Phys. JETP 38, 248 (1974)

Manakov system: Fundamental vector bright solitons

Fundamental bright-solitons: Single wavenumber → Single-hump profile

$$q_j = k_{1R}\hat{A}_j e^{i\eta_{1I}}\text{sech}(\eta_{1R} + \frac{R}{2}),\ \eta_{1R} = k_{1R}(t - 2k_{1I}z),\ \eta_{1I} = k_{1I}t + (k_{1R}^2 - k_{1I}^2)z, \tag{2}$$

$$\hat{A}_j = \alpha_1^{(j)}/[\sqrt{(|\alpha_1^{(1)}|^2 + |\alpha_1^{(2)}|^2)}],\ e^R = (|\alpha_1^{(1)}|^2 + |\alpha_1^{(2)}|^2)/(k_1 + k_1^*)^2,\ j = 1, 2.$$

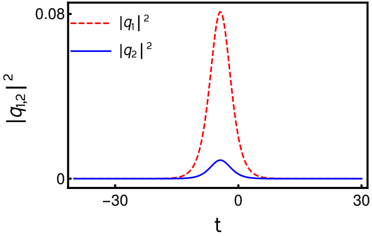

Figure: Degenerate one-soliton: $k_1 = 0.3 + 0.5i$, $\alpha_1^{(1)} = 1.5 + 1.5i$, $\alpha_1^{(2)} = 0.5 + 0.5i$.

R. Radhakrishnan and M. Lakshmanan, J. Phys. A: Math. Gen. 28, 2683 (1995)
R. Radhakrishnan, M. Lakshmanan and J. Hietarinta, Phys. Rev. E 56, 2213 (1997)
T. Kanna and M. Lakshmanan, Phys. Rev. Lett. 86, 5043 (2001)

Manakov system: Vector two bright soliton solution

Degenerate two bright soliton solution of Manakov system is

$$q_j = \frac{g^{(j)}}{f}, \quad j = 1, 2, \qquad (3)$$

where

$$g^{(j)} = \begin{vmatrix} A_{11} & A_{12} & 1 & 0 & e^{\eta_1} \\ A_{21} & A_{22} & 0 & 1 & e^{\eta_2} \\ -1 & 0 & B_{11} & B_{12} & 0 \\ 0 & -1 & B_{21} & B_{22} & 0 \\ 0 & 0 & -\alpha_1^{(j)} & -\alpha_2^{(j)} & 0 \end{vmatrix}, \qquad f = \begin{vmatrix} A_{11} & A_{12} & 1 & 0 \\ A_{21} & A_{22} & 0 & 1 \\ -1 & 0 & B_{11} & B_{12} \\ 0 & -1 & B_{21} & B_{22} \end{vmatrix}, \qquad (4)$$

in which $A_{ij} = \frac{e^{\eta_i + \eta_j^*}}{k_i + k_j^*}$, and $B_{ij} = \kappa_{ji} = \frac{\left(\alpha_j^{(1)}\alpha_i^{(1)*} + \alpha_j^{(2)}\alpha_i^{(2)*}\right)}{(k_j + k_i^*)}$, $\quad i, j = 1, 2.$

Manakov system: New collision property → Optical Computation

Transition intensities: $|T_j^1|^2 = \frac{|1-\lambda_2|^2}{|1-\lambda_1\lambda_2|}$, $|T_j^2|^2 = \frac{|1-\lambda_1\lambda_2|}{|1-\lambda_1|^2}$, $\lambda_1 = \frac{\kappa_{21}\alpha_1^{(j)}}{\kappa_{11}\alpha_2^{(j)}}$,
$\lambda_2 = \frac{\kappa_{12}\alpha_2^{(j)}}{\kappa_{22}\alpha_1^{(j)}}$, $\kappa_{ij} = \frac{(\alpha_i^{(1)}\alpha_j^{(1)*}+\alpha_i^{(2)}\alpha_j^{(2)*})}{(k_i+k_j^*)}$, $i,j = 1,2$.

Shape changing collision: $\frac{\alpha_1^{(1)}}{\alpha_2^{(1)}} \neq \frac{\alpha_1^{(2)}}{\alpha_2^{(2)}}$

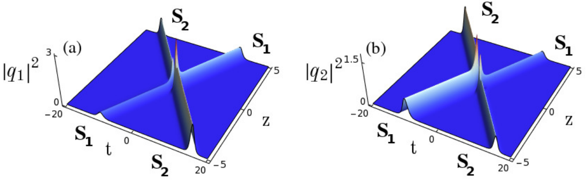

Figure: Energy sharing collision of two bright solitons in Manakov system: $k_1 = 1+i$, $k_2 = 1.51 - 1.51i$, $l_2 = 0.425 - 2.2i$, $\alpha_1^{(1)} = 0.5 + 0.5i$, $\alpha_2^{(1)} = \alpha_1^{(2)} = \alpha_2^{(2)} = 1$.

R. Radhakrishnan, M. Lakshmanan and J. Hietarinta, Phys. Rev. E 56, 2213 (1997)
T. Kanna and M. Lakshmanan, Phys. Rev. Lett. 86, 5043 (2001)

Optical Computing Applications

- It has been explained that the shape changing collision property of vector bright solitons provides the possibility of constructing logic gates.
- Such logic gates construction useful for all optical computing applications.

M. Vijayajayanthi, T. Kanna, K. Murali and M. Lakshmanan, *Phys. Rev. E* **97**, 060201(R) (2018);
M. H. Jakubowski, K. Steiglitz and R. Squier, *Phys. Rev. E* **58**, 6752 (1998);
K. Steiglitz, *Phys. Rev. E* **63**, 016608 (2000);
M.Soljacic, K. Steiglitz, S. M. Sears, M. Segev, M. H. Jakubowski, and R. Squier, Phys. Rev. Lett. **90**, 254102 (2003).

Manakov system: Elastic collision property

Elastic collision: $\frac{\alpha_1^{(1)}}{\alpha_2^{(1)}} = \frac{\alpha_1^{(2)}}{\alpha_2^{(2)}}$

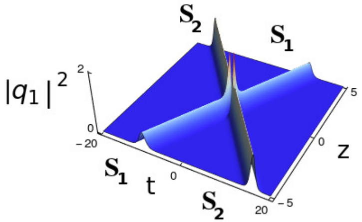

Figure: Elastic collision of two bright solitons in Manakov system: $k_1 = 1 + i$, $k_2 = 1.51 - 1.51i$, $l_2 = 0.425 - 2.2i$, $\alpha_1^{(1)} = \alpha_2^{(1)} = \alpha_1^{(2)} = \alpha_2^{(2)} = 1$.

R. Radhakrishnan, M. Lakshmanan and J. Hietarinta, Phys. Rev. E 56, 2213 (1997)
T. Kanna and M. Lakshmanan, Phys. Rev. Lett. 86, 5043 (2001)

Vector bright solitons in mixed 2-coupled NLS system

mixed 2-CNLS system

$$
\begin{aligned}
iq_{1,z} + q_{1,tt} + 2(|q_1|^2 - |q_2|^2)q_1 = 0 \\
iq_{2,z} + q_{2,tt} + 2(|q_1|^2 - |q_2|^2)q_2 = 0
\end{aligned} \quad (5)
$$

Nonsingular fundamental soliton: $|\alpha_1^{(1)}| > |\alpha_1^{(2)}|$

$$
q_j = k_{1R}\hat{A}_j e^{i\eta_{1I}} \mathrm{sech}(\eta_{1R} + \frac{R}{2}), \ \eta_{1R} = k_{1R}(t - 2k_{1I}z), \ \eta_{1I} = k_{1I}t + (k_{1R}^2 - k_{1I}^2)z, \quad (6)
$$

$$
\hat{A}_j = \alpha_1^{(j)}/[\sqrt{(|\alpha_1^{(1)}|^2 - |\alpha_1^{(2)}|^2)}], \ e^R = (|\alpha_1^{(1)}|^2 - |\alpha_1^{(2)}|^2)/(k_1 + k_1^*)^2, \ j = 1, 2.
$$

T. Kanna, M. Lakshmanan, P. T. Dinda and N. Akhmediev, *Phys. Rev. E* **73** (2006) 026604

Vector bright solitons in mixed 2-CNLS system

mixed 2-CNLS system: Shape changing collision

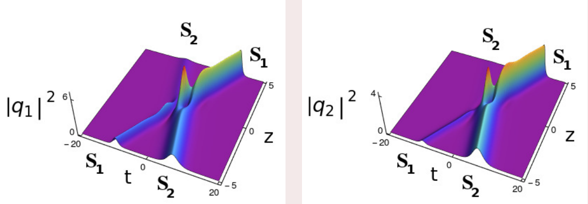

Figure: Energy sharing collision of two bright solitons in mixed 2-CNLS system: $k_1 = 1 + i$, $k_2 = 0.51 - 0.51i$, $\alpha_1^{(1)} = 1 + i$, $\alpha_2^{(1)} = 1 - i$, $\alpha_1^{(2)} = 0.5 - 0.5i$, $\alpha_2^{(2)} = 0.5$.

T. Kanna, M. Lakshmanan, P. T. Dinda and N. Akhmediev, *Phys. Rev. E* **73** (2006) 026604

Vector bright solitons in coherently coupled NLS system

Coherently coupled NLS system and its fundamental soliton solution:

$$iq_{1,z} - q_{1,tt} - \gamma(|q_1|^2 + 2|q_2|^2)q_1 - \gamma q_2^2 q_1^* = 0$$
$$iq_{2,z} - q_{2,tt} - \gamma(2|q_1|^2 + |q_2|^2)q_2 - \gamma q_1^2 q_2^* = 0 \quad (7)$$

$q_j = \frac{\alpha_j e^{\eta_1} + e^{2\eta_1+\eta_1^*+\delta_{1j}}}{1+e^{\eta_1+\eta_1^*+R_1}+e^{2(\eta_1+\eta_1^*)+\epsilon_{11}}}$, $j = 1, 2$, $\eta_1 = k_1 t - ik_1^2 z$, $e^{\delta_{11}} = \frac{\gamma\alpha_1^*(\alpha_1^2-\alpha_2^2)}{2(k_1+k_1^*)^2}$,
$e^{\delta_{12}} = -\frac{\gamma\alpha_2^*(\alpha_1^2-\alpha_2^2)}{2(k_1+k_1^*)^2}$, $e^{R_1} = \frac{\gamma(|\alpha_1|^2+|\alpha_2|^2)}{(k_1+k_1^*)^2}$, $e^{\epsilon_{11}} = \frac{\gamma^2(\alpha_1^2-\alpha_2^2)(\alpha_1^{2*}-\alpha_2^{2*})}{4(k_1+k_1^*)^4}$.

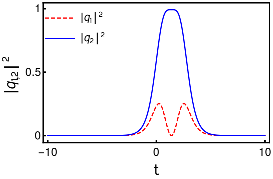

Figure: $k_1 = 1 + i, \gamma = 1$, $\alpha_1 = 0.71$, $\alpha_2 = 1$.

T. Kanna, M. Vijayajayanthi and M. Lakshmanan, *J. Phys. A: Math. Theor.* **43** (2010) 434018

Vector bright solitons in coherently coupled NLS system

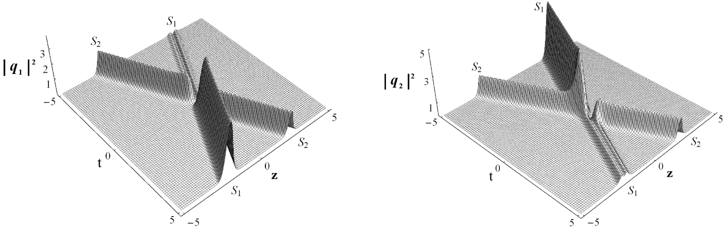

Figure: Shape changing collision of a coherently coupled soliton with incoherently coupled soliton: $\gamma = 2$, $k_1 = 2.3 + i$, $k_2 = 2.5i$, $\alpha_1 = 0.75$, $\beta_1 = 1.9$ and $\alpha_2 = \beta_2 = 3 + i$.

T. Kanna, M. Vijayajayanthi and M. Lakshmanan, *J. Phys. A: Math. Theor.* **43** (2010) 434018

Vector solitons in long-wave short-wave resonance interaction (LSRI) system

Two component Yajima-Oikawa system and its fundamental soliton solution

$$iS_t^{(1)} + S_{xx}^{(1)} + LS^{(1)} = 0,\ \ iS_t^{(2)} + S_{xx}^{(2)} + LS^{(2)} = 0,\ \ L_t = \sum_{l=1}^{2}(|S^{(l)}|^2)_x. \qquad (8)$$

$S^{(l)} = 2A_l k_{1R}\sqrt{k_{1I}}e^{i(\eta_{1I}+\frac{\pi}{2})}\text{sech}(\eta_{1R}+\frac{R}{2}),\ L = 2k_{1R}^2\text{sech}^2(\eta_{1R}+\frac{R}{2}),\ l = 1,2,$
$A_1 = \frac{\alpha_1}{(|\alpha_1|^2+|\beta_1|^2)^{1/2}},\ A_2 = \frac{\beta_1}{(|\alpha_1|^2+|\beta_1|^2)^{1/2}},\ \eta_{1R} = k_{1R}(t+2k_{1I}z),$
$\eta_{1I} = k_{1I}t + (k_{1R}^2 - k_{1I}^2)z,\ e^R = \frac{-(|\alpha_1|^2+|\beta_1|^2)}{16k_{1R}^2k_{1I}}.$

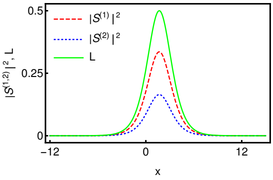

Figure: $k_1 = 0.5 - 0.5i,\ \alpha_1 = 0.5,\ \beta_1 = 0.35.$

Vector solitons in long-wave short-wave resonance interaction (LSRI) system

Two component Yajima-Oikawa system: Shape changing collision

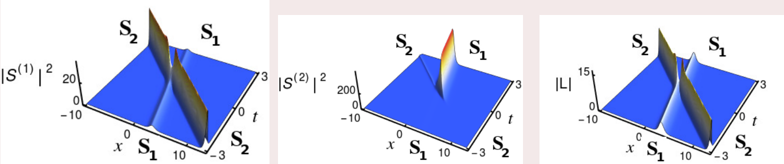

Figure: $k_1 = 1.3 - 0.5i$, $k_2 = 2.2 - 2i$, $\alpha_1^{(1)} = 2.5$, $\alpha_2^{(1)} = 1.3$, $\alpha_1^{(2)} = 0.8$, $\alpha_2^{(2)} = 0.6$.

T. Kanna, K. Sakkaravarthi and K. Tamilselvan, *Phys. Rev. E* **88** (2013) 062921;
T Kanna, M Vijayajayanthi, K Sakkaravarthi, M Lakshmanan, *J. Phys. A: Math. Theor.* **42** (2009) 115103

Already konwn class of vector bright solitons: Degenerate

Degeneracy

- It is clear from the above studies that the above mentioned degeneracy in wave numbers always persists in the previously reported vector bright solitons.
- Such vector bright solitons always exhibit single-hump structure only (in CCNLS system double-hump structure only observed - not more than a double-hump profile) .

Motivation

Based on the nature of presence of wave numbers in the multi-component soliton solution we classify them as

Degenerate soliton

The solitons which propagate in all the modes with identical wave numbers designated as degenerate solitons.

Nondegenerate soliton

The solitons which propagate in all the modes with non-identical wave numbers referred as non-degenerate solitons.

Motivation

- Do solitons exist with non-identical wave numbers in all the modes? $\rightarrow$ Yes they exist!
- What will happen if the fundamental solitons described by non-identical wave numbers? $\rightarrow$ Multi-hump structure solitons will emerge!
- What is the nature of the collision scenario?

Bilinearization of Manakov equation

- To bilinearize the Manakov equation, we consider the bilinear transformation $q_j = \frac{g^{(j)}(z,t)}{f(z,t)}$, $j = 1, 2$.
- Bilinear forms are

$$\begin{aligned}(iD_z + D_t^2) g^{(j)} \cdot f &= 0, j = 1, 2, \\ D_t^2 f \cdot f &= 2 \sum_{n=1}^{2} g^{(n)} g^{(n)*}.\end{aligned}$$

- Hirota's bilinear operators D_z and D_t are defined as

$$D_z^n D_t^m (a.b) = \left(\frac{\partial}{\partial z} - \frac{\partial}{\partial z'}\right)^n \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^m a(z,t) b(z',t')\Big|_{(z=z',t=t')}$$

- In general to construct soliton solutions one has to solve the bilinear forms with $g^{(j)} = \epsilon g_1^{(j)} + \epsilon^3 g_3^{(j)} + \ldots$, $j = 1, 2$, and $f = 1 + \epsilon^2 f_2 + \epsilon^4 f_4 + \ldots$

Nondegenerate fundamental solitons

To find out the exact analytical form of fundamental soliton solution,

- we consider the seed solutions as $g_1^{(1)} = \alpha_1^{(1)} e^{\eta_1}$ and $g_1^{(2)} = \alpha_1^{(2)} e^{\xi_1}$, respectively, to the resultant linear partial differential equations $(iD_z + D_t^2) g_1^{(j)} \cdot 1 = 0$, $j = 1, 2$, which arise in the lowest order of ϵ.
- Here, $\eta_1 = k_1 t + ik_1^2 z$, $\xi_1 = l_1 t + il_1^2 z$, and $\alpha_1^{(j)}$, $j = 1, 2$, k_1 and l_1 are in general independent complex wave numbers.
- For the above choice of seed solutions, the series expansion gets terminated for fundamental soliton solution as $g^{(j)} = \epsilon g_1^{(j)} + \epsilon^3 g_3^{(j)}$ and $f = 1 + \epsilon^2 f_2 + \epsilon^4 f_4$.

S. Stalin, R. Ramakrishnan, M. Senthilvelan and M. Lakshmanan, Phys. Rev. Lett. **122**, 043901 (2019)

Nondegenerate fundamental solitons

Nondegenerate fundamental soliton

A new class of fundamental soliton solution is obtained as

$$q_1 = (\alpha_1^{(1)} e^{\eta_1} + e^{\eta_1+\xi_1+\xi_1^*+\Delta_1^{(1)}})/D_1$$
$$q_2 = (\alpha_1^{(2)} e^{\xi_1} + e^{\eta_1+\eta_1^*+\xi_1+\Delta_1^{(2)}})/D_1. \qquad (9)$$

Here $D_1 = 1 + e^{\eta_1+\eta_1^*+\delta_1} + e^{\xi_1+\xi_1^*+\delta_2} + e^{\eta_1+\eta_1^*+\xi_1+\xi_1^*+\delta_{11}}$, $e^{\Delta_1^{(1)}} = \frac{(k_1-l_1)\alpha_1^{(1)}|\alpha_1^{(2)}|^2}{(k_1+l_1^*)(l_1+l_1^*)^2}$, $e^{\Delta_1^{(2)}} = -\frac{(k_1-l_1)|\alpha_1^{(1)}|^2\alpha_1^{(2)}}{(k_1+k_1^*)^2(k_1^*+l_1)}$, $e^{\delta_1} = \frac{|\alpha_1^{(1)}|^2}{(k_1+k_1^*)^2}$, $e^{\delta_2} = \frac{|\alpha_1^{(2)}|^2}{(l_1+l_1^*)^2}$ and $e^{\delta_{11}} = \frac{|k_1-l_1|^2|\alpha_1^{(1)}|^2|\alpha_1^{(2)}|^2}{(k_1+k_1^*)^2(k_1^*+l_1)(k_1+l_1^*)(l_1+l_1^*)^2}$.

Degenerate soliton

In the degenerate limit, $(k_1 = l_1)$, the above solution can be reduced as

$$q_j = k_{1R}\hat{A}_j e^{i\eta_{1I}} sech(\eta_{1R} + \frac{R}{2}). \qquad (10)$$

The other constants that are appearing in the above are defined in Eq. (2).

Properties of Nondegenerate one soliton solution

Various symmetric profiles

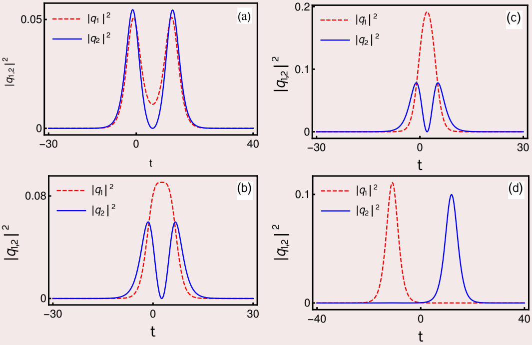

Figure: Various symmetric intensity profiles of nondegenerate fundamental solitons: (a): $k_1 = 0.333 + 0.5i$, $l_1 = 0.315 + 0.5i$, $\alpha_1^{(1)} = 0.45 + 0.45i$, $\alpha_1^{(2)} = 0.49 + 0.45i$. (b): $k_1 = 0.425 + 0.5i$, $l_1 = 0.3 + 0.5i$, $\alpha_1^{(1)} = 0.44 + 0.51i$, $\alpha_1^{(2)} = 0.43 + 0.5i$. (c): $k_1 = 0.55 + 0.5i$, $l_1 = 0.333 + 0.5i$, $\alpha_1^{(1)} = 0.5 + 0.5i$, $\alpha_1^{(2)} = 0.5 + 0.45i$. (d): $k_1 = 0.333 + 0.5i$, $l_1 = -0.316 + 0.5i$, $\alpha_1^{(1)} = 0.45 + 0.5i$, $\alpha_1^{(2)} = 0.5 + 0.5i$.

Properties of Nondegenerate one soliton solution

Various asymmetric profiles

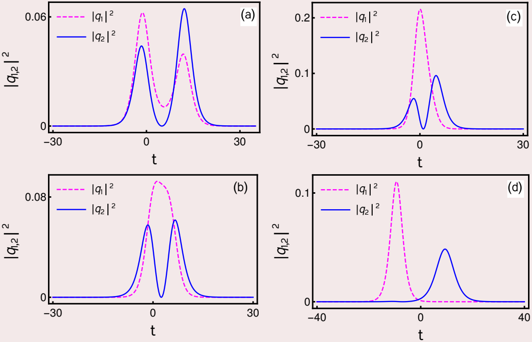

Figure: Various asymmetric intensity profiles of nondegenerate fundamental solitons: (a): $k_1 = 0.333 + 0.5i, l_1 = 0.315 + 0.5i$, $\alpha_1^{(1)} = 0.65 + 0.45i$, $\alpha_1^{(2)} = 0.49 + 0.45i$. (b): $k_1 = 0.425 + 0.5i, l_1 = 0.3 + 0.5i$, $\alpha_1^{(1)} = 0.5 + 0.51i$, $\alpha_1^{(2)} = 0.43 + 0.5i$. (c): $k_1 = 0.55 + 0.5i, l_1 = 0.333 + 0.5i$, $\alpha_1^{(1)} = 1.2 + 0.5i$, $\alpha_1^{(2)} = 0.5 + 0.45i$. (d): $k_1 = 0.333 + 0.5i, l_1 = -0.22 + 0.5i$, $\alpha_1^{(1)} = 0.45 + 3i$, $\alpha_1^{(2)} = 0.5 + 0.5i$.

Properties of Nondegenerate one soliton solution

Double-hump formation: Relative velocity $\Delta v = v_1 - v_2 \to 0$

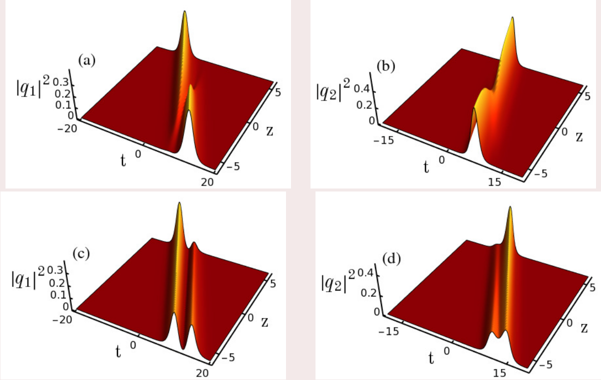

Figure: (a) and (b) represent the node formation in soliton profiles: $k_1 = 0.65 - 0.85i$, $l_1 = 0.78 - 0.5i$, $\alpha_1^{(1)} = 1$ and $\alpha_1^{(2)} = 0.5$;
(c) and (d) denote the emergence of double-hump in both the modes: $k_1 = 0.65 - 0.85i$, $l_1 = 0.78 - 0.8i$, $\alpha_1^{(1)} = 1$ and $\alpha_1^{(2)} = 0.5$.

Nondegenerate two-soliton solution

Nondegenerate two-soliton solution of Manakov system is $q_N = \frac{g^{(N)}}{f}$,

$$g^{(N)} = \begin{vmatrix} A & I & \phi \\ -I & B & \mathbf{0}^T \\ \mathbf{O} & C_N & 0 \end{vmatrix}, \quad f = \begin{vmatrix} A & I \\ -I & B \end{vmatrix}, \quad N = 1, 2. \tag{11}$$

Here the matrices A and B are defined as

$$A = \begin{pmatrix} A_{mm'} & A_{mn} \\ A_{nm} & A_{nn'} \end{pmatrix}, \quad B = \begin{pmatrix} \kappa_{mm'} & \kappa_{mn} \\ \kappa_{nm} & \kappa_{nn'} \end{pmatrix}, \tag{12}$$

The various elements of matrix A are obtained from the following matrix elements,

$$A_{mm'} = \frac{e^{\eta_m + \eta^*_{m'}}}{(k_m + k^*_{m'})}, \ A_{mn} = \frac{e^{\eta_m + \xi^*_n}}{(k_m + l^*_n)}, A_{nn'} = \frac{e^{\xi_n + \xi^*_{n'}}}{(l_n + l^*_{n'})}, \ A_{nm} = \frac{e^{\eta^*_n + \xi_m}}{(k^*_n + l_m)}, \tag{13}$$

The elements of matrix B are defined as

$$\kappa_{mm'} = \frac{\psi^\dagger_m \sigma \psi_{m'}}{(k^*_m + k_{m'})}, \ \kappa_{mn} = \frac{\psi^\dagger_m \sigma \psi'_n}{(k^*_m + l_n)}, \ \kappa_{nm} = \frac{\psi'^\dagger_n \sigma \psi_m}{(l^*_n + k_m)}, \ \kappa_{nn'} = \frac{\psi'^\dagger_n \sigma \psi'_{n'}}{(l^*_n + l_{n'})}, \tag{14}$$

$$m, m', n, n' = 1, 2.$$

Nondegenerate two-soliton solution

$$\psi_j = \begin{pmatrix} \alpha_j^{(1)} \\ 0 \end{pmatrix}, \ \ \psi'_j = \begin{pmatrix} 0 \\ \alpha_j^{(2)} \end{pmatrix}, j = m, m' n, n' = 1, 2.$$

The other matrices in Eq. (42) are defined below:

$\phi = \begin{pmatrix} e^{\eta_1} & e^{\eta_2} & e^{\xi_1} & e^{\xi_2} \end{pmatrix}^T$, $C_1 = -\begin{pmatrix} \alpha_1^{(1)} & \alpha_2^{(1)} & \alpha_3^{(1)} & 0 & 0 & 0 \end{pmatrix}$,

$C_2 = -\begin{pmatrix} 0 & 0 & 0 & \alpha_1^{(2)} & \alpha_2^{(2)} & \alpha_3^{(2)} \end{pmatrix}$, $\mathbf{O} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

and $\sigma = I$ is a $(n \times n)$ identity matrix.

Elastic collision of nondegenerate solitons

Shape preserving collision of symmetric solitons

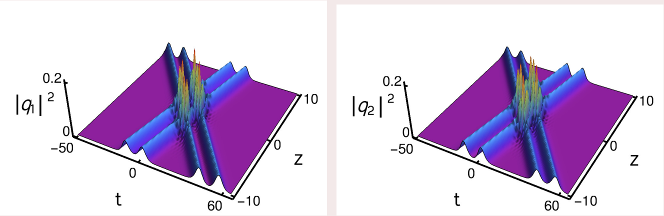

Figure: Shape preserving collision between two symmetric double-hump solitons: $k_1 = 0.333 + 0.5i$, $l_1 = 0.315 + 0.5i$, $k_2 = 0.315 - 2.2i$, $l_2 = 0.333 - 2.2i$, $\alpha_1^{(1)} = 0.45 + 0.45i$, $\alpha_2^{(1)} = 0.49 + 0.45i$, $\alpha_1^{(2)} = 0.49 + 0.45i$ and $\alpha_2^{(2)} = 0.45 + 0.45i$.

- flattop and double-hump solitons
- single-hump-double-hump solitons
- two single-hump solitons

Elastic collision of nondegenerate solitons

Shape preserving collision of asymmetric solitons

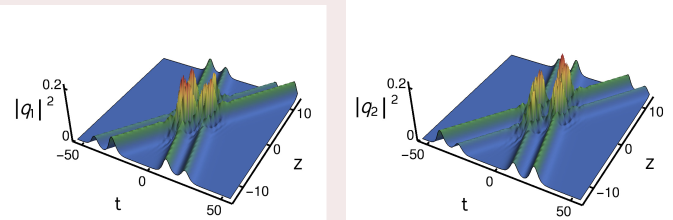

Figure: shape preserving collision between two asymmetric solitons: $k_1 = 0.333 - 0.5i$, $l_1 = 0.315 - 0.5i$, $k_2 = 0.315 + 1.5i$, $l_2 = 0.333 + 1.5i$, $\alpha_1^{(1)} = 0.65 + 0.45i$, $\alpha_2^{(1)} = 0.49 + 0.5i$, $\alpha_1^{(2)} = 0.49 + 0.5i$ and $\alpha_2^{(2)} = 0.65 + 0.45i$

Elastic collision of nondegenerate solitons

Shape preserving collision between symmetric and asymmetric nondegenerate solitons

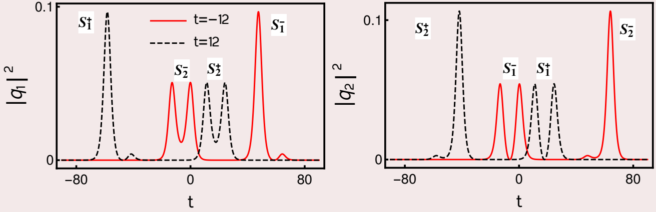

Figure: Shape preserving collision between symmetric double-hump soliton and asymmetric double-hump soliton: $k_1 = 0.333 + 0.5i$, $l_1 = 0.315 + 0.5i$, $k_2 = 0.315 - 2.2i$, $l_2 = 0.333 - 2.2i$, $\alpha_1^{(1)} = 0.45 + 0.45i$, $\alpha_2^{(1)} = 2.49 + 2.45i$, $\alpha_1^{(2)} = 0.49 + 0.45i$ and $\alpha_2^{(2)} = 0.45 + 0.45i$.

Elastic collision of nondegenerate solitons

Shape altering collision

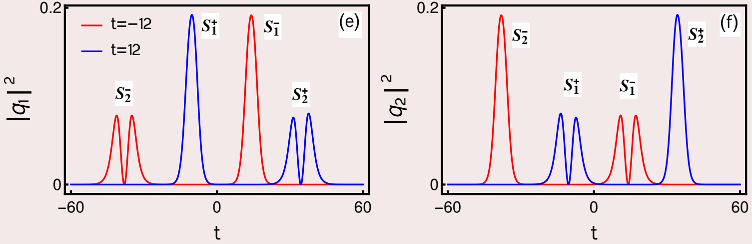

Figure: (e) and (f) denote shape altering collision between symmetry double-hump soliton and symmetry single-hump soliton: $k_1 = 0.41 + 0.5i$, $l_1 = 0.305 + 0.5i$, $k_2 = 0.305 - 2.2i$, $l_2 = 0.41 - 2.2i$, $\alpha_1^{(1)} = \alpha_2^{(2)} = 0.44 + 0.499i$ and $\alpha_2^{(1)} = \alpha_1^{(2)} = 0.44 + 0.5i$

Elastic collision of nondegenerate solitons

Shape altering collision

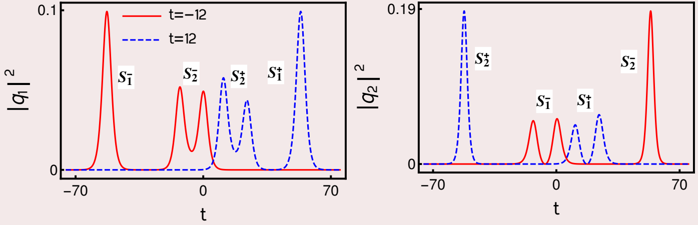

Figure: Shape altering collision between asymmetry double-hump soliton and symmetry single-hump soliton: $k_1 = 0.41 + 0.5i$, $l_1 = 0.305 + 0.5i$, $k_2 = 0.305 - 2.2i$, $l_2 = 0.41 - 2.2i$, $\alpha_1^{(1)} = \alpha_2^{(2)} = 0.44 + 0.499i$ and $\alpha_2^{(1)} = \alpha_1^{(2)} = 0.44 + 0.5i$

Confirmation of elastic collision

- To confirm the nature of collision, we have carried out asymptotic analysis. From the obtained asymptotic forms we have calculated the following transition amplitudes and transition intensities.
- Transition amplitudes are calculated as $T_j^l = \frac{A_j^{l+}}{A_j^{l-}}$, $j, l = 1, 2$
- Transition intensity is defined as $|T_j^l|^2$, $l, j = 1, 2$.
- Here $|T_j^l|^2 = 1$, $l, j = 1, 2 \rightarrow$ it corresponds to elastic collision.
- Total intensity of each of the solitons is conserved.
  $|A_j^{l-}|^2 = |A_j^{l+}|^2$, $j, l = 1, 2$
- Total intensity in each of the modes is also conserved.
  $|A_j^{1-}|^2 + |A_j^{2-}|^2 = |A_j^{1+}|^2 + |A_j^{2+}|^2 =$ constant.

Shape changing collision: Near degenerate limit

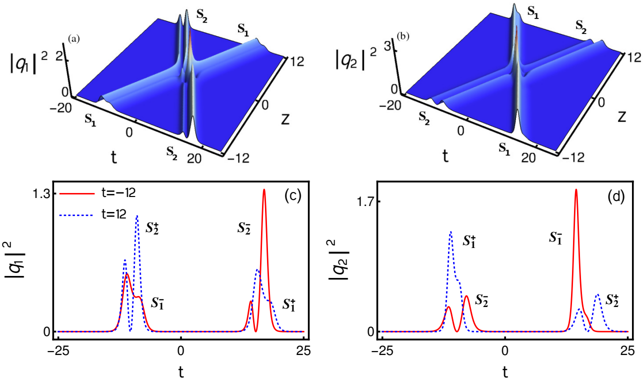

Figure: Shape changing collision between two asymmetric double-hump solitons: $k_1 = 1.2 - 0.5i$, $l_1 = 0.8 + 0.5i$ $k_2 = 1.0 + 0.5i$, $l_2 = 1.5 - 0.5i$, $\alpha_1^{(1)} = \alpha_2^{(2)} = 0.5 + 0.5i$, $\alpha_2^{(1)} = \alpha_1^{(2)} = 0.45 + 0.5i$.

Collision between degenerate soliton and nondegenerate soliton

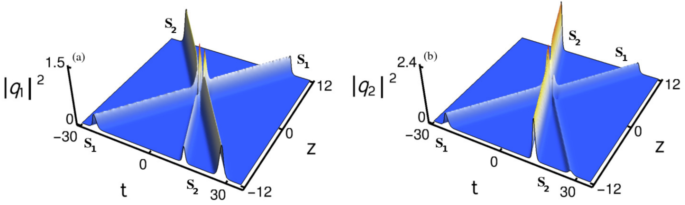

Figure: Shape changing collision between degenerate and nondegenerate solitons: $k_1 = l_1 = 1 + i$, $k_2 = 1 - i$, $l_2 = 1.5 - 0.5i$, $\alpha_1^{(1)} = 0.8 + 0.8i$, $\alpha_2^{(2)} = 0.6 + 0.6i$, $\alpha_2^{(1)} = 0.25 + 0.25i$, $\alpha_1^{(2)} = 1 + i$.

R. Ramakrishnan, S. Stalin and M. Lakshmanan, *Nondegenerate solitons and their collisions in Manakov system*, (Submitted for Publication).

Collision between degenerate soliton and nondegenerate soliton

- The asymptotic analysis reveals that energy redistribution occurs between modes q_1 and q_2. In order to confirm the inelastic nature of this interesting collsion process we obtain the following expression for the transition amplitudes,

$$
\begin{aligned}
T_1^1 &= \frac{(k_1 - l_2)^{1/2}(k_1^* + l_2)(k_1^* + k_2)C_1^{1/2}}{(k_1^* - l_2^*)^{1/2}(k_1 + k_2^*)(k_1 + l_2^*)C_2^{1/2}}, \\
T_2^1 &= \frac{(k_1 - k_2)^{1/2}(k_1 - l_2)(k_1^* + k_2)^{1/2}(k_1^* + l_2)C_1^{1/2}}{(k_1^* - k_2^*)^{1/2}(k_1^* - l_2^*)(k_1 + k_2^*)^{1/2}(k_1 + l_2^*)C_3^{1/2}}, \qquad (15)
\end{aligned}
$$

where $C_1 = |\alpha_1^{(1)}|^2 + |\alpha_1^{(2)}|^2$, $C_2 = |\alpha_1^{(1)}|^2 + \chi|\alpha_1^{(2)}|^2$,
$C_3 = (|\alpha_1^{(1)}|^2/\chi) + |\alpha_1^{(2)}|^2$,
$\chi = [(k_1^* - k_2^*)|k_1 - l_2|^2|k_1 + k_2^*|^2]/[(k_1 - k_2)^3(k_1 + l_2^*)^2]$.

- If the quantity T_j^l is not unimodular (for this case the constant $\chi \neq 1$) then the degenerate and nondegenerate solitons always exhibit inelastic collision.
- The standard elastic collision can be recovered when $\chi = 1$.

Nondegenerate fundamental soliton in 3-CNLS system

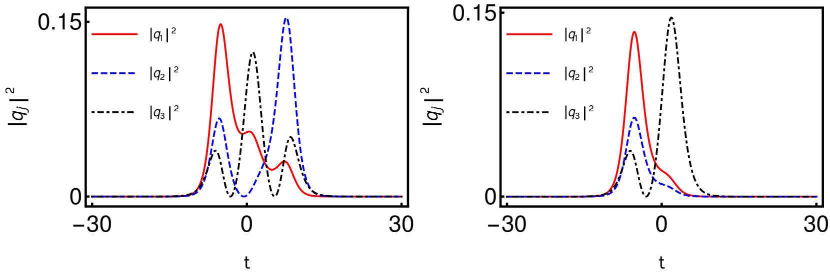

Figure: (i) Asymmetric triple-hump profile: $k_1 = 0.53 + 0.5i$, $k_2 = 0.5 - 0.5i$, $k_3 = 0.4 + 0.5i$, $\alpha_1^{(1)} = 0.65 + 0.65i$, $\alpha_2^{(1)} = 0.45 - 0.45i$, $\alpha_3^{(1)} = 0.35 + 0.35i$. (ii) Asymmetric double-hump profile: $k_1 = k_2 = 0.5 + 0.5i$, $k_3 = 0.4 + 0.5i$, $\alpha_1^{(1)} = 0.65 + 0.65i$, $\alpha_2^{(1)} = 0.45 - 0.45i$, $\alpha_3^{(1)} = 0.35 + 0.35i$.

R. Ramakrishnan, S. Stalin, and M. Lakshmanan, *Nondegenerate multi-hump fundamental solitons in N-coupled nonlinear Schrödinger equations* (under preparation).

Nondegenerate fundamental soliton in 4-CNLS system

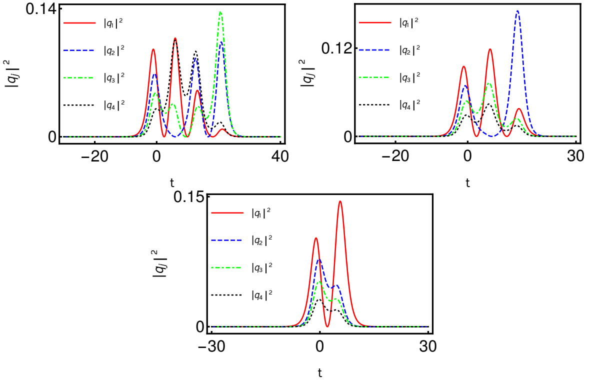

Figure: (i) Asymmetric quadruple-hump profile: $k_1 = 0.45 + 0.5i$, $k_2 = 0.5 + 0.5i$, $k_3 = 0.525 + 0.5i$, $k_4 = 0.55 + 0.5i$, $\alpha_1^{(1)} = 0.65 + 0.65i$, $\alpha_2^{(1)} = 0.55 - 0.55i$, $\alpha_3^{(1)} = 0.45 + 0.45i$, $\alpha_4^{(1)} = 0.35 - 0.35i$. (ii) Asymmetric triple-hump profile: $k_1 = 0.45 + 0.5i$, $k_2 = 0.5 + 0.5i$, $k_3 = k_4 = 0.55 + 0.5i$, $\alpha_1^{(1)} = 0.65 + 0.65i$, $\alpha_2^{(1)} = 0.55 - 0.55i$, $\alpha_3^{(1)} = 0.45 + 0.45i$, $\alpha_4^{(1)} = 0.35 - 0.35i$. (ii) Asymmetric double-hump profile: $k_1 = 0.45 + 0.5i$, $k_2 = k_3 = k_4 = 0.55 + 0.5i$, $\alpha_1^{(1)} = 0.65 + 0.65i$, $\alpha_2^{(1)} = 0.55 - 0.55i$, $\alpha_3^{(1)} = 0.45 + 0.45i$, $\alpha_4^{(1)} = 0.35 - 0.35i$.

Nondegenerate fundamental soliton in mixed 2-CNLS system

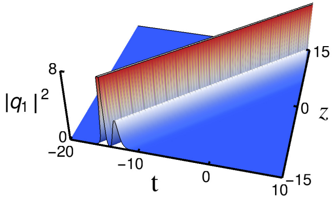

Figure: Singular double-hump profile: $k_1 = 1.2 + 0.5i$, $l_1 = -0.5 + 0.5i$, $\alpha_1 = 0.3$ and $\beta_1 = i$.

S. Stalin, R. Ramakrishnan and M. Lakshmanan, *Nondegenerate soliton solutions in certain coupled nonlinear Schrödinger systems*, Phys. Lett. A, 384, 126201 (2020)

Nondegenerate fundamental soliton in coherently coupled NLS system

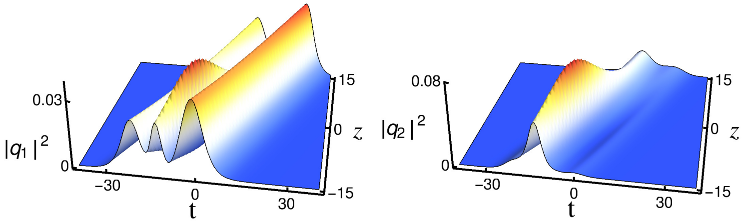

Figure: Breathing type triple-hump profile: $\gamma = 2$, $k_1 = 0.21 + 0.5i$, $l_1 = 0.29 + 0.5i$, $\alpha_1 = 0.95 + 0.5i$ and $\beta_1 = 0.97 - i$.

Nondegenerate fundamental soliton in LSRI system/2-component YO system

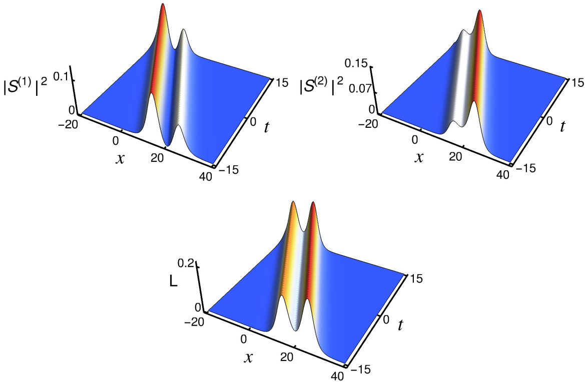

Figure: Asymmetric double-hump profile: $k_1 = 0.3 - 0.5i$, $l_1 = 0.35 - 0.5i$, $\alpha_1 = 0.8$, $\beta_1 = 0.5$.

Conclusions

- Coupled NLS systems continue to bring out novel and surprising features.
- Nondegenerate and degenerate soliton interactions give rise to fascinating behaviours.
- Much work needs to be carried out to understand these systems fully.