

Homoclinic Explosion in a Lorenz Like System

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We study the transition to Chaos in a Lorenz like system describing the dynamics of cell signaling. By tracing the branch of limit cycles past a limit point with the aid of numerical bifurcation analysis we reveal the existence of a homoclinic bifurcation point. The existence of the symmetric pair of the homoclinic trajectories is proved analytically with the use of undetermined coefficient method. We show the existence of a homoclinic explosion that gives rise first to a preturbulence regime and then to a transition to a stable chaotic regime.

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