# Steady States and Traveling Waves of Heisenberg and M-I Spin Systems

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We first review our recent results on the steady states and traveling wave solutions of the Heisenberg and M-I spin systems [1] expressed as 1+1 and 1+2 PDEs respectively in the form: $\vec{S}_t = \vec{S} \times \vec{S}_{xx}$, and $\vec{S}_t = (\vec{S} \times \vec{S}_y + u\vec{S})_x$, $u_x = -(\vec{S}, \vec{S}_x \times \vec{S}_y)$, $\vec{S} = (S_1, S_2, S_3)$, $S_1^2+S_2^2+S_3^2 = 1$, Reducing these equations to systems of ODEs, we have shown that they can be solved analytically in terms of simple trigonometric functions on the unit sphere. We also present new results on the steady states and traveling wave solutions of the Landau-Lifshitz-Gilbert (LLG) equation for Heisenberg spins (see [2],[3])

$$\vec{S}_t = \vec{S} \times \vec{S}_{xx} + \lambda(\vec{S}_{xx} - (\vec{S} \cdot \vec{S}_{xx})\vec{S}), \quad \vec{S} = (u, v, w), u^2 + v^2 + w^2 = 1,$$

where $\lambda$ is a Gilbert damping parameter. We point out that these solutions represent stable attractors that would be interesting to investigate further in terms of their basin of attraction.

**References**

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[2] M. Lakshmanan, "The fascinating world of the Landau-Lifshitz-Gilbert equation: An overview", *Phil. Trans. R. Soc.* **A 369**, 1280-1300 (2011).

[3] M. Lakshmanan, M. Daniel, "Perturbation of solitons in the classical continuum isotropic Heisenberg spin system" *Physica* **A 120**, 125-152 (1983).