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Transformation Rules for Elastic Scattering Coefficients

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Project. The concept of scattering coefficients has played a pivotal role in a broad range of inverse scattering and imaging problems in acoustic, electromagnetic, and elastic media. In view of their promising applications in inverse problems related to mathematical imaging [1] and invisibility cloaking, the notion of elastic scattering coefficients was presented in [2, 3]. These objects are somehow linked to the matrix theory for scattering of elastic waves by obstacles of various geometric nature (e.g. cracks, inclusions, cavities, etc.). The incident and scattered fields both admit multipolar expansions in terms of cylinderical or spherical wave functions owing to Jacobi-Anger decomposition and wave addition theorems. The coefficients of the expansion of the field scattered by a given obstacle are connected to those of the incident field by an infinite matrix, whose elements are termed as *scattering coefficients*. Such a matrix is independent of the choice of incident field, and depends only on the morphology of the obstacles and the frequency of incidence.

In this capstone project, we intend to discuss the transformation rules for elastic scattering coefficients in two and three dimensions. The impetus behind this study is a promising application of the elastic scattering coefficients in gesture-based input/output devices. The transformation properties (translation, rotation, and scaling) of elastic scattering coefficients are important to determine some shape invariants and design shape descriptors in elastic media. Once the invariants and descriptors are known, one can design a protocol for matching the elastic scattering coefficients of the user-defined input gestures to those of the gestures predefined in a dictionary and finally, decide an appropriate output response accordingly. Therefore, this capstone project in which we focus on the transformation rules is the first step towards designing a gesture-based input/output device.

The results of this capstone project may lead to a publication.

Pre-requisites: Basic knowledge of wave propagation and elasticity, excellent command on linear algebra and tensor algebra, strong analytic skills.

References

[1] H. Ammari, E. Bretin, J. Garnier, H. Kang, H. Lee, and A. Wahab: *Mathematical Methods in Elasticity Imaging* (Princeton University Press, 2015).

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[3] H. Liu, W. Y. Tsui, and A. Wahab: Three-dimensional elastic scattering coefficients and enhancement of the elastic near cloaking, submitted to *J. Elast.*, Mar. 2020, (arXiv:2003.10725).