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Reconstruction of Elastic Scattering Coefficients from Multi-static Response Measurements in 3D

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Project. The concept of scattering coefficients has played a pivotal role in a broad range of inverse scattering and imaging problems in acoustic, electromagnetic, and elastic media. In view of their promising applications in inverse problems related to mathematical imaging [1] and invisibility cloaking, the notion of elastic scattering coefficients was presented in [2, 3]. These objects are somehow linked to the matrix theory for scattering of elastic waves by obstacles of various geometric nature (e.g. cracks, inclusions, cavities, etc.). The incident and scattered fields both admit multipolar expansions in terms of cylinderical or spherical wave functions owing to Jacobi-Anger decomposition and wave addition theorems. The coefficients of the expansion of the field scattered by a given obstacle are connected to those of the incident field by an infinite matrix, whose elements are termed as *scattering coefficients*. Such a matrix is independent of the choice of incident field, and depends only on the morphology of the obstacles and the frequency of incidence.

In this capstone project, we intend to design and analyze a least-squares based framework for reconstructing the elastic scattering coefficients of an unknown inclusion from multi-static response data in 3D. In fact, these frequency dependent mathematical objects contain rich information about the contrast of material parameters, high-order shape oscillations, frequency profile, and the maximal resolving power of an imaging setup. Therefore, if one can reconstruct the elastic scattering coefficients of an inclusion from the measurements of the scattered wave-fields, one can deal with a number of inverse scattering problems. This capstone project is the first step towards designing an inverse scattering framework for elastic media.

The results of this capstone project may lead to a publication.

Pre-requisites: Excellent command on linear algebra and tensor algebra, basic knowledge of wave propagation and elasticity, and strong analytic skills.

References

[1] H. Ammari, E. Bretin, J. Garnier, H. Kang, H. Lee, and A. Wahab: *Mathematical Methods in Elasticity Imaging* (Princeton University Press, 2015).

[2] T. Abbas, H. Ammari, G. Hu, A. Wahab, and J. C. Ye: Two-dimensional elastic scattering coefficients and enhancement of nearly elastic cloaking, *J. Elast.*, 128:(2017), pp. 203-243.

[3] H. Liu, W. Y. Tsui, and A. Wahab: Three-dimensional elastic scattering coefficients and enhancement of the elastic near cloaking, submitted to *J. Elast.*, Mar. 2020, (arXiv:2003.10725).