Capstone Project descriptions

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I suggest the following directions for Capstone Projects. Any of these could lead to a publication. I am also open to discussing any other research opportunities. My students are usually hired as research assistants by our research grants and received travel support for international conferences/workshops or/and internships.

Topic 1. **Geometric functional inequalities and applications**

We will deal with a fascinating area of mathematical analysis devoted to functional inequalities associated with operators with different geometries. This is an internationally very active area of research that witnessed a big boost during the last years. In this project, we will aim at deriving new inequalities and at linking them to several problems of geometry and physics, as well as the analysis on (homogeneous) Lie groups.

Topic 2. **Spectral geometry of partial differential equations**

The aim of this research project is to analyze the studies underpinning spectral geometric inequalities arising in the theory of partial differential equations (PDE). We study modern new techniques of the relevant spectral theory, the advanced techniques of geometric rearrangements, and systematically demonstrate the learned techniques in various applications ranging from MEMS (micro-electro mechanical systems) problems to problems of galactic dynamics.

Topic 3. **Nonlocal models**

The basic idea of this project is to do qualitative analysis for general nonlocal and fractional models as well as to study their consequences and applications. Thus, the main purpose is to construct new methods for qualitative analysis for general nonlocal and fractional models. Our unifying methods can be readily used to recover most of the previously known results as well as to construct other new techniques which will be useful to study/understand new nonlocal and fractional models in different fields.

Topic 4. **Inverse problems for subelliptic evolution equations**

In this project, we aim to obtain the solvability/non-solvability results to direct and inverse problems for evolution equations for general homogeneous invariant subelliptic differential operators on general graded Lie groups. Examples of graded Lie groups include the Euclidean group $\mathbb{R}^n$, the Heisenberg group, and general stratified Lie groups. The considered class of Lie groups is the most general class of nilpotent Lie groups where one can still consider hypoelliptic homogeneous invariant differential operators and the corresponding subelliptic differential equations. The examples are the pseudo-parabolic equations for the sub-Laplacian on the Heisenberg group or on general stratified Lie groups, or p-evolution equations for higher order operators, already in all these cases our results will be new. We will also study nonlinear cases, that is when the source function is nonlinear. In the nonlinear case, blow-up/nonexistence results can be obtained.

Topic 5. **Partial differential equations with singularities**

We will deal with different models of partial differential equations with coefficients exhibiting singular behavior. It is well known that the classical theory of distributions does not apply in the case of strong singularities, however, recently new approaches have emerged based on the so-called very weak solutions. We will investigate the properties of such solutions in several fundamental models from points of view of both pure and applied mathematics.