Capstone Project descriptions

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January 12, 2024

1 Method of heat polynomials for solving inverse Stefan type problems

In this project, we will deal with application of heat polynomials method for solving one-dimensional inverse and direct problems. It is required to systematize the known properties of one-dimensional heat polynomials and it is also necessary to supplement them with new properties that will allow to use them for solving problems in areas with moving boundaries, as well as to evaluate the error of the approximate solution. With the help of the constructed heat polynomials the solution of classical one- and two-phase heat conduction problems with a free boundary at an unknown heat flux will be obtained, for the solution of which the method of collocation heat polynomials is supposed to be used.

2 Method of heat polynomials for solving inverse two-dimensional Stefan problems

The aim of this research project is develop the method of heat polynomials with its application to solve direct and inverse heat transfer problems in spatial multidimensional and axisymmetric conditions, including phase transformations. However, the practical implementation of the method of heat polynomials for the exact analytical solution of the two-dimensional Stefan problem faces the fundamental difficulty of obtaining recurrence formulas for the determination of unknown coefficients. Therefore, the methods presented in [1, 2, 3, 4] for one-dimensional inverse Stefan type problems will be extended to the cases of two-dimensional problems.

3 Solution of Stefan type problems using Deep Learning

In this capstone project, we intend to extend the physics-informed neural networks (PINNs) [5, 6] for different type of Stefan type problems. In particular, the PINNs will be applied to inverse Stefan-type problems with unknown heat source function. Another Stefan problem, which is planned to be investigated in the framework of the project, is related to heat and mass transfer in electrical contacts.

Bibliografia

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