Capstone Project(Evolutionary differential equations)
Supervisor: Amin Esfahani
Office: 7.238
Email: amin.esfahani@nu.edu.kz

My primary research interests lie in the study of evolutionary differential equations, with a background in other areas such as Physics, Chemistry, Engineering, etc. When faced with specific evolution equations emerging from the modeling process, a crucial aspect of mathematical analysis is establishing the correct formulation of the problem.

The evolution of a system, depending on a continuous/discrete time variable t, is described by an equation of the form $\dot{u} = f(t, u)$, where the dot denotes a time derivative, $u(t) \in X$ is the state of the system at time t, and f is a given vector field on X. The space X is the state space of the system; a point in X specifies the instantaneous state of the system. We assume that X is a Banach space. When X is finite-dimensional, the evolution equation is a system of ordinary differential equations (ODE's). Some Partial differential equations (PDE's) can be regarded as evolution equations on an infinite-dimensional state space. Higher-order equations can be written as first-order equations by introducing time derivatives as new dependent variables.

Many models are presented by evolution equations, such as the Heat equation governing temperature distribution in an object, the wave equation describing waves (e.g., water waves, sound waves, and seismic waves), or the KdV equation modeling waves on shallow water surfaces. In Fluid mechanics, the motion of a viscous incompressible fluid is modeled by an evolution equation, known as the incompressible Navier-Stokes or Euler equations. The Schrödinger equation is another well-known evolution equation governing the wave function of a quantum-mechanical system.

Various questions can be asked about evolution equations, including the existence of particular types of solutions like equilibrium solutions, travelling waves, self-similar solutions, time-periodic solutions; the dynamic stability of these solutions; the long-time asymptotic behavior of solutions; chaotic dynamics; complete integrability; singular perturbation expressions for solutions; evolution of random solutions; convergence of numerical schemes; and more. To address each question, one needs to be familiar with the literature and related tools and techniques.

Research Opportunities:

1. **Equilibrium Solutions:**
   - Investigate the existence and properties of equilibrium solutions in evolving systems.
2. **Travelling Waves and Self-similar Solutions:**

- Explore patterns and behaviors of travelling waves and self-similar solutions.

3. **Dynamical systems:**

   - Study the stability of evolutionary systems and their dynamical behavior.

4. **Long-term Asymptotic Behavior:**

   - Examine the behavior of solutions over extended periods, revealing insights into system dynamics.

5. **Chaotic Dynamics:**

   - Chaotic behavior and its implications in evolutionary systems.

6. **Complete Integrability:**

   - Explore systems where solutions can be expressed in terms of elementary functions.

7. **Singular Perturbation Expressions:**

   - Analyze solutions in regimes with significant changes, offering insights into system transitions.

8. **Evolution of Random Solutions:**

   - Investigate the influence of randomness on evolutionary systems.

9. **Convergence of Numerical Schemes:**

   - Examine the reliability and accuracy of numerical methods for solving evolution equations.